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1. The ground and low-lying non-rotational states of medium and heavy nuclei are rather well described by means of quasiparticles and phonons. With increasing excitation energy the level density strongly increases and the structure of the states becomes more complicated due to the interaction of quasiparticles with phonons, the Coriolis interaction and others /1/. The wave functions of the states with energies much higher than the magnitude of the gap have many components containing different number of quasiparticles. The high excited states are very complicated and their density is rather large. Therefore, they are usually described with the nuclear statistical model. The name high excited states is given to states with excitation energies close to the nucleon binding energy and still higher.

It is interesting to study the structure of the high excited states by means of semi-microscopic description using the quasiparticle and phonon operators. Then the following problems arise. What is the contribution of two-, four- and so on quasiparticle components to the wave function of the high excited state? Do neighboring states with identical spins I and parities π differ strongly from one another?

There are experimental evidence that neighboring high excited states have different structure. So, there is a noticeable difference of gamma-ray spectra obtained from resonances with identical I^{π} and very close energies. The experimental data /2/ on $(n\alpha)$ reactions testify that the reduced probabilities of alpha decays into the ground and excited states strongly change when passing from one resonance to another. Some information on the high excited states can be obtained from the study of the $(n\alpha)$-reactions on thermal neutrons /3/.

It is very interesting to clarify what information on some components of the wave functions can be obtained from experimental investigations of $(n\alpha)$ and $(n\gamma)$ reactions on resonances. The present paper is just devoted to the calculation of the reduced probabilities of alpha and gamma transitions from the high excited states to the ground and lowlying states and to their analysis.

2. We construct the wave function of the high excited state of an even-even nucleus basing on the semi-microscopic approach. The nuclear wave function is the product of the wave function of proton and neutron systems each of which consists of zero, two, four and so on quasiparticle terms. Since the main contribution of the zero quasiparticle component is contained in the ground state of an even-even nucleus, the wave function of a high excited state is represented in the form of a superposition of two, four and so on quasiparticle states. We do not introduce the operators of the quadrupole and octupole phonons. The latter are a superposition of the two-quasiparticle terms and therefore, we assume that they enter the general expansion.

We construct the wave function of the excited state for even-even spherical nucleus. It is difficult to make a generalization to the case of the even-even deformed nucleus and to the case of odd and odd-odd nuclei. The one-particle state is characterized by the total momentum j and its projection m as well as by other quantum numbers of the isotopic spin, N and ℓ which are not given here in an explicit form.

We represent the wave function in the form of a product of the quasiparticle operators

$$\alpha^+_{jm} = u_j a^+_{jm} + (-1)^{j-m} v_j a_{j-m} , \tag{1}$$

acting on the wave function of the quasiparticle vacuum

$$\Psi_0 = \prod_{j,m>0} (u_j + (-1)^{j-m} v_j a^+_{jm} a^+_{j-m}) \Psi_{00} , \tag{2}$$

where $a_{jm}\Psi_{00} = 0$, a_{jm} is the nucleon absorption operator. The coefficients of the Bogolubov canonical transformation are

$$v_j^2 = \frac{1}{2}\{ 1 - \frac{E(j) - \lambda}{\epsilon(j)} \}, \quad u_j^2 = 1 - v_j^2 ,$$

where $E(j)$ is the single-particle energy, $\epsilon(j) = \sqrt{C^2 + (E(j) - \lambda)^2}$, C is the correlation function, λ is the chemical potential (see, ref. /4/).

The wave function of the high excited state must contain the two-quasiparticle components, which in the case $j_1 m_1 \neq j_2 m_2$ are written in the form

$$\alpha^+_{j_1 m_1} \alpha^+_{j_2 m_2} \Psi_0 = a^+_{j_1 m_1} a^+_{j_2 m_2} \Psi_0 (j_1 m_1 , j_2 m_2), \tag{3}$$

where

$$\Psi_0(j_1 m_1, j_2 m_2) = \prod_{\substack{j,m>0 \\ jm \neq j_1 m_1 ;\, j_2 m_2}} (u_j + (-1)^{j-m} v_j a^+_{jm} a^+_{j-m}) \Psi_{00} . \qquad (4)$$

Among the two-quasiparticle components there are proton $2p$ and neutron $2n$ components of three types: particle-particle, particle-hole and hole-hole.

The strength of the two-quasiparticle state is distributed over a number of levels with an energy close to the energy $\epsilon(j_1) + \epsilon(j_2)$. The two-quasiparticle components, the energies of which $\epsilon(j) + \epsilon(j')$ are much lower than the energy of the state under consideration, have already distributed their strength over the levels near $\epsilon(j) + \epsilon(j')$ and do not noticeably contribute to the high excited states. At high excitation energy the strength of the two-quasiparticle state is distributed over a large number of levels and therefore, as a rule, the fraction of each of them is not large.

The wave function of the high excited state, for the exception of the two-quasiparticle components, contains four, six and so on quasiparticle components of the type

$$a^+_{j_1 m_1} a^+_{j_2 m_2} \ldots a^+_{j_k m_k} a^+_{j_{k+1} m_{k+1}} \ldots a^+_{j_\ell m_\ell} \times$$
$$\times \Psi_0(N; j_1 m_1, \ldots, j_k m_k) \Psi_0(Z; j_{k+1} m_{k+1} \ldots j_\ell m_\ell). \qquad (5)$$

Here there are k neutron production operators and $\ell - k$ proton production operators, k and ℓ being even.

Among four-quasiparticle components there are neutron $4n$ and neutron-proton $2n\,2p$ and proton $4p$. The four-quasiparticle components with an energy $\epsilon(j_1)+\epsilon(j_2)+\epsilon(j_3)+\epsilon(j_4)$ close to the energy of the state under consideration contribute to the wave function of the high excited state. If at an excitation energy of 3-4 MeV there are levels to the wave functions of which one two-quasiparticle component contribute greatly, then at an energy of 6-8 MeV there must be states which have not small contribution of the appropriate component $2n\,2p$. Similar considerations may be extended to six and so on quasiparticle components.

Among the products $(a^+_{jm}\; a^+_{j-m})$ of two operators with identical j there are such which have $I=0$. It is known[4] that among the two-quasiparticle 0^+ states there is one spurious state and their wave functions are mutually nonorthogonal. On order that the complexes $(a^+_{jm}\; a^+_{j-m})_{I=0}$ will not damage the wave function of the high excited state, instead them we introduce the phonon operators of pairing vibrations. The phonon operators determined separately for the neutron and the proton systems have the following form:

$$\Omega^+_i(t) = \frac{1}{2}\sum_{j,m} \frac{(-1)^{j-m}}{\sqrt{2j+1}} \{ X^i(j)\, a^+_{jm}\, a^+_{j-m} - Y^i(j)\, a_{j-m}\, a_{jm} \}, \qquad (6)$$

where i is the number of the root of the corresponding secular equation, and the functions $X^i(j)$ and $Y^i(j)$ are given, e.g. in ref.[5]; $t=n$ denotes the neutron and $t=p$ the proton systems.

If in the wave function there are components containing the products operator of quasiparticles and phonons then the corresponding states in phonon $\Omega_i(t)$ should be blocked. For example, in the product $a^+_{j_1m_1}\, a^+_{j_2m_2}\, \Omega^+_i(t)$ containing only neutron operators instead of $\Omega^+_i(t)$ in the form (6) we should take the operator

$$\Omega_{i}^{+}(t;j_1 m_1, j_2 m_2) =$$

$$= \frac{1}{2} \sum_{\substack{j,m \\ jm \neq j_1 m_1 ; j_2 m_2}} \frac{(-1)^{j-m}}{\sqrt{2j+1}} \{ X^{i}(j) a_{jm}^{+} a_{j-m}^{+} - Y^{i}(j) a_{j-m} a_{jm} \} . \tag{7}$$

Thus, the wave function of the high excited state with $I \neq 0$ of an even-even spherical nucleus can be written in the form

$$\Psi(I^{\pi} M) = \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_{t} b_{IM}^{2t}(j_1 m_1, j_2 m_2) a_{j_1 m_1}^{+} a_{j_2 m_2}^{+} \Psi_0^{2t}(j_1 m_1 ; j_2 m_2) +$$

$$+ \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}}{}' \sum_{t,t'} b_{IM}^{2t,2t'}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) a_{j_1 m_1}^{+} a_{j_2 m_2}^{+} a_{j_3 m_3}^{+} a_{j_4 m_4}^{+} \times$$

$$\times \Psi_0^{2t,2t'}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) + \tag{8}$$

$$+ \sum_{\substack{j_1, j_2, j_3, j_4, j_5, j_6 \\ m_1, m_2, m_3, m_4, m_5, m_6}}{}' \sum_{t,t'} b_{IM}^{4t,2t'}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4, j_5 m_5, j_6 m_6) a_{j_1 m_1}^{+} a_{j_2 m_2}^{+} a_{j_3 m_3}^{+} \times$$

$$\times a_{j_4 m_4}^{+} a_{j_5 m_5}^{+} a_{j_6 m_6}^{+} \cdot \Psi_0^{4t,2t'}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4, j_5 m_5, j_6 m_6) + \ldots +$$

$$+\sum_{\substack{j_1,j_2\\ m_1,m_2}}{}' \sum_{t,t' i} b_{IM}^{2t,\Omega_i(t')}(j_1 m_1, j_2 m_2)\, a^+_{j_1 m_1} a^+_{j_2 m_2} \Omega_i^+(t', j_1 m_1, j_2 m_2) \times$$

$$\times \Psi_0^{2t}(j_1 m_1, j_2 m_2) + \sum_{\substack{j_1,j_2,j_3,j_4\\ m_1,m_2,m_3,m_4}}{}' \sum_{t,t',t''} b_{IM}^{2t,2t',\Omega_i(t'')}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) \times \tag{8}$$

$$\times a^+_{j_1 m_1} a^+_{j_2 m_2} a^+_{j_3 m_3} a^+_{j_4 m_4} \Omega_i^+(t''; j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) \times$$

$$\times \Psi_0^{2t,2t'}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) + \cdots + \sum_{\substack{j_1,j_2\\ m_1,m_2}}{}' \sum_{t,t',t'' i i'} b_{IM}^{2t,\Omega_i(t'),\Omega_i(t'')}(j_1 m_1, j_2 m_2)\, a^+_{j_1 m_1} a^+_{j_2 m_2} \times$$

$$\times \Omega_i^+(t'; j_1 m_1, j_2 m_2)\, \Omega_{i'}^+(t''; j_1 m_1, j_2 m_2)\, \Psi_0^{2t}(j_1 m_1, j_2 m_2) + \ldots .$$

The summation $\sum'_{j_1, j_2, \ldots ;\, m_1, m_2, \ldots}$ means that the terms $j_1 = j_2$, $|m_1| = |m_2|$ are absent and than $E(j_1) < E(j_2) \leq \ldots$. The index $t = n$ indicates the neutron and $t = p$ the proton systems;

$$\Psi_0^{2n}(j_1 m_1, j_2 m_2) = \Psi_0(N; j_1 m_1, j_2 m_2)\, \Psi_0(Z), \tag{9}$$

$$\Psi_0^{2n2n}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) = \Psi_0(N; j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4)\Psi_0(Z),$$

$$\Psi_0^{2n,2p}(j_1 m_1, j_2 m_2; j_3 m_3, j_4 m_4) = \tag{9}$$

$$= \Psi_0(N; j_1 m_1, j_2 m_2)\Psi_0(Z; j_3 m_3, j_4 m_4)$$

and so on.

The coefficients $b_{IM}^{2t,2t'}(j_1 m_1, j_2 m_2 \dots)$ in (8) define the contribution of the corresponding quasiparticle component, they contain the products of the Clebsh-Gordon coefficients. The normalization condition of the wave function (8) is of the form:

$$(\Psi^*(I^\pi M)\Psi(I^\pi M) = 1 = \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_t |b_{IM}^{2t}(j_1 m_1, j_2 m_2)|^2 +$$

$$+ \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}}{}' \sum_{t,t'} |b_{IM}^{2t,2t'}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4)|^2 + \dots + \tag{10}$$

$$+ \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_{t,t',i} |b_{IM}^{2t,\Omega_i(t')}(j_1 m_1, j_2 m_2)|^2 + \dots .$$

To ensure the conservation of the particle number on the average it is necessary for each term in (8) to solve the equations for the correlation functions C and the chemical potentials λ taking into account the blocking effect.

The wave function of the high excited state with $I^{\pi} = 0^{+}$ contains the phonon operators $\Omega_{I}^{+}(t; jm, \ldots)$ and the products $(a^{+}_{j_1 m_1} a^{+}_{j_2 m_2} \ldots)$ for which $I^{\pi} = 0^{+}$.

It should be noted that the expression (8) for the wave function of the high excited state is not the most general one. In constructing the wave function use is made of the approximation consisting in that in each term the operators of quasiparticles and the pairing vibration phonons act on the wave function being the quasiparticle vacuum.

3. We consider the decay of the high excited state with $I \neq 0$ to the ground state of the daughter even-even spherical nucleus.

First of all we give the main formulas of the alpha decay in the formulation suggested in ref. /6/. The alpha decay probability is written in the form

$$w_{I} = \Sigma \gamma^{2}_{I\ell} P_{\ell}, \tag{11}$$

where P_ℓ is the penetrability through the potential barrier and $\gamma^{2}_{I\ell}$ is the reduced width. The quantity $\gamma_{I\ell}$ is of the form

$$\gamma_{I\ell} = (\Psi^{*}_{f}(N-2, Z-2) \frac{1}{4} \sum_{\substack{j_p, j'_p, j_n, j'_n \\ m_p, m'_p, m_n, m'_n}} W(j_p m_p, j'_p m'_p \mid j_n m_n, j'_n m'_n) \times \tag{12}$$

$$\times a_{j_p m_p} a_{j'_p m'_p} a_{j_m m_m} a_{j'_n m'_n} \Psi_{i}(N, Z)).$$

The function $W(j_p m_p, j'_p m'_p \mid j_n m_n, j'_n m'_n)$ describes the probability of production of an alpha particle from protons in the states $j_p m_p$, $j'_p m'_p$ and from neutrons on the states $j_n m_n, j'_n m'_n$. The alpha decay is treated as a single-stage process.

For the alpha transitions between the ground states of even-even nuclei

$$\gamma_{00} = \sum_{j_p, j_n} W(j_p - m_p, j_p m_p \mid j_n - m_n, j_n m_n) \times$$

$$\times (-1)^{j_p - m_p} (-1)^{j_n - m_n} \quad u'_{j_p} v_{j_p} u'_{j_n} v_{j_n}, \tag{13}$$

where u_j, v_j are related to the parent and u'_j, v'_j to the daughter nuclei.

In (13) and in the subsequent formulas the multipliers of the type

$$\prod_j (u_j u'_j + v_j v'_j), \tag{14}$$

which are somewhat smaller than unity are rejected.

The alpha transition probabilities between the ground states of even-even nuclei are considerably enhanced by summing in (13) over the neutron and proton system levels. To estimate this enhancement it is supposed that the diagonal terms W with $j_p = j'_p$, $m_p = -m'_p$ or with $j_n = j'_n$, $m_n = -m'_n$ are independent of $j_p m_p$ or of $j_n m_n$. Then

$$\sum_j u'_j v_j \approx \sum_j u'_j v'_j = \frac{C}{G} \approx 6 - 8, \tag{15}$$

G is the pairing interaction constant. After squaring we get that in the neutron and proton system the transition enhacement is 50-60 which leads to an increase of the alpha transition probability between the ground states of nonmagic nuclei by a factor of $(2\text{-}4)10^3$.

We calculate the function γ for the alpha transition from the high excited state described with the wave function (8) to the ground state of an even-even nucleus described with the wave function (2). As a result we obtain

$$\gamma\,(I^{\pi} M \to 0^{+}) = \sum_{\substack{j_1, j_2\\ m_1, m_2}}{}' \sum_{t} b^{2t}_{IM}(j_1 m_1, j_2 m_2) \times$$

$$\times \sum_{j, m>0} W\,(j-m, jm \mid j_2 m_2, j_1 m_1)(-1)^{j-m}\, v_j\, u'_j\, u'_{j_1}\, u'_{j_2} +$$

$$+ \sum_{\substack{j_1, j_2, j_3, j_4\\ m_1, m_2, m_3, m_4}} b^{2n,2p}_{IM}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4)\, W\,(j_4 m_4, j_3 m_3 \mid j_2 m_2, j_1 m_1) \times \qquad (16)$$

$$\times\, u'_{j_1}\, u''_{j_2}\, u'_{j_3}\, u'_{j_4} + \sum_{\substack{j_1, j_2\\ m_1, m_2}} \sum_{\substack{t, t', i\\ t \neq t'}} b^{2t, \Omega_i(t')}_{IM}(j_1 m_1, j_2 m_2) \times$$

$$\times \sum_{j, m>0} W\,(j-m, jm \mid j_2 m_2, j_1 m_1) \frac{(-1)^{j-m}}{\sqrt{2j+1}} (X^{i}(j)\, u^2_j - Y^{i}(j)\, v^2_j)\, u'_{j_1}\, u'_{j_2}\,.$$

From (16) it is seen that the contribution of the two-quasiparticle components $2n$ and $2p$ of the particle-particle type is essentially enhanced. To the alpha decay probability the contribution is also came from the four-quasiparticle components $2n\,2p$ and the components two quasiparticles and a phonon $2n\Omega_i(p)$ and $2p\,\Omega_i(n)$ due to multipliers u'_j only the particle states being involved.

If a high excited state has $I^{\pi} = 0^{+}$ then instead of the first term in (16) we have

$$\sum_{t,i} b_0^{\Omega_i(t)} \sum_{\substack{j,j' \\ m>0,\, m'>0}} W\,(j-m, jm \mid j' -m', j'm') \times \tag{17}$$

$$\times (-1)^{j-m} v_j u_{j'} \frac{(-1)^{j'-m'}}{\sqrt{2j'+1}} (X^i(j') u_{j'}^2 - Y^i(j') v_{j'}^2).$$

It is seen from the equation that any one-phonon component of the wave function of the high excited state contributes to the probability of the alpha decay to the ground state. Therefore the fluctuations of the reduced widths for transitions to the ground states from resonances $I^{\pi} = 0^{+}$ must be smaller compared with the transitions from resonances $I \neq 0$.

4. We consider the alpha decay of a high excited state to the one-phonon and the two-quasiparticle states of an even-even spherical nucleus.

The wave function of the first $(i = 1)$ one-phonon state of the daughter nucleus is taken in the form

$$Q_1^{+}(\lambda\mu)\Psi,$$

where the phonon operator is

$$Q_1^{+}(\lambda\mu) = \frac{1}{2}\sum_{j,j'} \{\psi_{jj'}^{\lambda 1} A^{+}(j,j';\lambda\mu) - (-1)^{\lambda-\mu} \phi_{jj'}^{\lambda 1} A(j,j';\lambda-\mu)\},$$

$$A\,(j,j';\lambda\mu) = \sum_{m,m'} (j\, m, j'm' \mid \lambda\mu) a_{j'm'} a_{jm}, \tag{18}$$

the functions $\psi_{jj'}^{\lambda 1}$ and $\phi_{jj'}^{\lambda 1}$ are determined in, e.g. ref./7/.

The matrix element of the alpha decay from the high excited $I \neq 0$ state to the first one-phonon state is of the form

$$\gamma(I^{\pi}M \to \lambda\mu) = \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_t b^{2t}_{IM}(j_1 m_1, j_2 m_2)\{\psi^{\lambda\mu}_{j_1 j_2} \cdot (j_1 m_1 j_2 m_2|\lambda\mu) \times$$

$$\times \sum_{\substack{j,j' \\ m>0, m'>0}} W(j'-m', j'm'|j-m, jm)(-1)^{j'-m'} v_{j'} u'_{j'} (-1)^{j-m} v_j u'_j +$$

$$+ \frac{1}{2} u'_{j_1} u_{j_2} \sum_{\substack{j,j' \\ m,m'}} W(jm, j'm'|j_2 m_2, j_1 m_1)(\psi^{\lambda 1}_{jj'} (j-m j'-m'|\lambda\mu) \times$$

$$\times (-1)^{j-m} v_j (-1)^{j'-m'} v_{j'} - \phi^{\lambda 1}_{jj'} (jmj'm'|\lambda\mu) u_j u_{j'}) + \frac{1}{2} \sum_{\substack{j,j' \\ m,m'>0}} (-1)^{j'-m'} \times \quad (19)$$

$$\times v_{j'} u'_{j'} [W(j'-m', j'm'|jm, j_1 m_1)(-1)^{j-m} v_j u_{j_1} \psi^{\lambda 1}_{jj_2} (j-m j_2 m_2|\lambda\mu) -$$

$$- W(j'-m', j'm'|jm, j_2 m_2)(-1)^{j-m} v_j u_{j_2} \psi^{\lambda 1}_{jj'} (j-m j_1 m_1|\lambda\mu)]\} +$$

$$+ \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}}{}' b^{2n,2p}_{IM} (j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4)\{u'_{j_1} u'_{j_2} \psi^{\lambda 1}_{j_3 j_4} (j_3 m_3 j_4 m_4|\lambda\mu).$$

$$\cdot \sum_{j,m>0} W\,(j-m,\, jm \mid j_2 m_2,\; j_1 m_1)\, v_j u'_j\, (-1)^{j-m} + \ldots \} +$$

$$+ \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}}{}' \; \sum_t b^{4t}_{IM}\,(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) \sum_{j,m>0} (-1)^{j-m}\, v_j u'_j \;\times$$

$$\times \{ W\,(j-m,\, jm \mid j_4 m_4,\; j_3 m_3)\, u'_{j_4} u'_{j_3}\; \psi^{\lambda 1}_{j_1 j_2}\,(j_1 m_1\, j_2 m_2 \mid \lambda\mu) + \ldots \} +$$

$$+ \sum_{\substack{j_1, j_2, j_3, j_4, j_5, j_6 \\ m_1, m_2, m_3, m_4, m_5, m_6}}{}' \; \sum_{t,t'} b^{4t,2t'}_{IM}\,(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4, j_5 m_5, j_6 m_6)\, u'_{j_5} u'_{j_6} \times$$

$$\times \;\{ W\,(j_6 m_6,\; j_5 m_5 \mid j_4 m_4,\; j_3 m_3)\, u'_{j_4} u'_{j_3}\; \psi_{j_1 j_2}\,(j_1 m_1\, j_2 m_2 \mid \lambda\mu) + \ldots \} + \ldots .$$

Besides the mentioned ones there are terms in which two quasi-particle operators are replaced by phonon Ω^+_i and the four quasi-particle operators are replaced by the two phonons $\Omega^+_i\, \Omega^+_{i'}$.

The number of the components of the wave function (8) which contribute to the alpha decay probability essentially increases in the alpha decay to the one-phonon states compared with the decay to the ground state. In the alpha transition to the one-phonon state there may be involved the following types of the components of the wave function (8):

$2n; 2p; 2n2p; 4n; 4p; 4n2p; 2n4p; 2n\Omega_i(p);$

$2p\Omega_i(n); 2n2p\Omega_i(n); 2n2p\Omega_i(p); 4n\Omega_i(p); 4p\Omega_i(n);$

$2n\Omega_i(n)\Omega_{i'}(p); 2p\Omega_i(n)\Omega_{i'}(p); 2n\Omega_i(p)\Omega_{i'}(p); 2p\Omega_i(n)\Omega_{i'}(n).$

The alpha transitions to the one-phonon states from those two-quasiparticle components for which $\psi^{\lambda 1}_{j_1 j_2} \neq 0$ are enhanced; and the contribution of the components of the particle-particle type is somewhat weakened compared with the transition to the ground state. The contribution of the components of the type $2n2p$ for which $\psi^{\lambda 1}_{j_1 j_2} \neq 0$ and $u'_{j_3} u'_{j_4}$ is close to unity is somewhat enhanced. Only those components of the type $4n$ or $4p$ contribute to the probability of the alpha transition to the one-phonon state for which the product $\psi^{\lambda 1}_{j_1 j_2} u'_{j_3} u'_{j_4}$ differs from zero.

The reduced alpha widths for transition from the high excited 0^+ state to the one-phonon 0^+ ones having the character of pairing vibrations have not any remarkable feature. This is due to the fact that it is unlikely that the phonon Ω_i^+ with $i=1$ contributes noticeably to the one-phonon part of the wave function (8).

We calculate the function γ for the alpha transition from a high excited state to a two-quasiparticle neutron (for definiteness) state described with the wave function

$$\Psi(I_f M_f | j_f, j_f') = \sum_{m_f, m_f'} (j_f m_f | j_f' m_f' | I_f M_f) a^+_{j_f m_f} a^+_{j_f' m_f'} \Psi_0 . \qquad (20)$$

As a result we have

$$\gamma\,(I^{\pi}M \to I_f(j_f\,,j_f')) = \sum_{m_f,m_f'} (j_f m_f\, j_f' m_f' \mid I_f M_f) \times$$

$$\times \Big\{ \sum_{\substack{j_f, j_2 \\ m_1, m_2}}{}' \; b^{2n}_{IM}(j_1 m_1, j_2 m_2) \big[\delta_{j_1 j_f} \delta_{m_1 m_f} \delta_{j_2 j_f'} \delta_{m_2 m_f'} \times$$

$$\times \sum_{\substack{j_p, j \\ m_p>0,\, m>0 \\ jm \neq j_f m_f;\; j_f' m_f'}} W(j_p - m_p, j_p m_p \mid j - m, jm)\, u'_{j_p} (-1)^{j_p - m_p} v_{j_p} u_j' (-1)^{j-m} v_j - \tag{21}$$

$$- \cdots - \delta_{j_1 j_f} \delta_{m_1 m_f} \sum_{j_p, m_p>0} W(j_p - m_p, j_p m_p \mid j_f' m_f', j_2 m_2) u'_{j_p} (-1)^{j_p - m_p} v_{j_p} \times$$

$$\times (-1)^{j_f' - m_f'} v_{j_f'} u'_{j_2} - \cdots \big] + \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \; b^{2p}_{IM}(j_1 m_1, j_2 m_2)\, 4\, W(j_2 m_2, j_1 m_1 \mid j_f - m_f, j_f' - m_f') \times$$

$$\times u'_{j_1} u'_{j_2} (-1)^{j_f - m_f} v_{j_f} (-1)^{j_f' - m_f'} v_{j_f'} + \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}}{}' \; b^{2n,2p}_{IM}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) \times$$

$$\times [\delta_{j_1 j_f} \delta_{m_1 m_f} \delta_{j_2 j_f'} \delta_{m_2 m_f'} \sum_{\substack{j,m>0 \\ jm \neq j_f m_f; j_f' m_f'}} W(j_3 m_3, j_4 m_4 | j-m, jm) u_{j_3}' u_{j_4}' u_j' \times$$

$$\times (-1)^{j-m} v_j - \dots - \delta_{j_1 j_f} \delta_{m_1 m_f} W(j_3 m_3, j_4 m_4 | j_f' m_f', j_f m_f) u_{j_3}' u_{j_4}' \times$$

$$\times (-1)^{j_f' - m_f'} v_{j_f'} u_{j_2}' + \dots] + \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}}{}' b_{IM}^{4n} (j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) \times$$

$$\times [\delta_{j_3 j_f} \delta_{m_3 m_f} \delta_{j_4 j_f'} \delta_{m_4 m_f'} \sum_{j,m>0} W(j-m, jm | j_2 m_2, j_1 m_1) u_j' (-1)^{j-m} \times \tag{21}$$

$$\times v_j u_{j_1}' u_{j_2}' + \dots] + \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_i b_{IM}^{2p, \Omega_i(n)} (j_1 m_1, j_2 m_2) u_{j_1}' u_{j_2}' \times$$

$$\times [W(j_1 m_1, j_2 m_2 | j_f' - m_f', j_f - m_f) \frac{(-1)^{j_f - m_f}}{\sqrt{2j_f + 1}} X^i (j_f) u_{j_f} (-1)^{j_f' - m_f'} v_{j_f'} +$$

$$+ \dots] + \dots \} .$$

In the alpha decay of the high excited state to the two-quasiparticle state of the daughter nucleus some rigid selection rules are imposed on the components of the wave function (8). They consist in the following. One or two quasiparticles in (8) must be in the same subshells as the quasiparticles in the final state do. Due to these selection rules the reduced probabilities of alpha transitions to the two-quasiparticle states must be smaller than those to the one-phonon states.

5. We consider the electromagnetic transitions from the high excited states described with the wave function (8) to the ground one-phonon and two-quasiparticle state of even-even spherical nuclei.

The operators of $E\lambda$ and $M\lambda$ transitions are written in the form

$$\mathfrak{M}(E\lambda) = \frac{-1}{\sqrt{2\lambda+1}} \sum_{j,j'} \langle j'|\Gamma(E\lambda)|j\rangle (-1)^{j+j'-\lambda} \{ v^{(-)}_{jj'} B(j,j';\lambda\mu) + \frac{1}{2} u^{(+)}_{jj'} [A^{+}(j,j';\lambda\mu) + (-1)^{\lambda-\mu} A(j,j';\lambda-\mu)]\}, \tag{22}$$

$$\mathfrak{M}(M\lambda) = \frac{1}{\sqrt{2\lambda+1}} \sum_{j,j'} \langle j'|\Gamma(M\lambda)|j\rangle (-1)^{j+j'-\lambda} \{ v^{(+)}_{jj'} B(j,j';\lambda\mu) + \frac{1}{2} u^{(-)}_{jj'} [A^{+}(j,j';\lambda\mu) - (-1)^{\lambda-\mu} A(j,j';\lambda-\mu)]\}, \tag{23}$$

where

$$B(j,j';\lambda\mu) = \sum_{m,m'} (-1)^{j'+m'} (j m j' m' | \lambda\mu) a^{+}_{jm} a_{j'-m'}, \tag{24}$$

$$u^{(\pm)}_{jj'} = u_j v_{j'} \pm u_{j'} v_j , \quad v^{(\pm)}_{jj'} = u_j u_{j'} \pm v_j v_{j'} . \tag{25}$$

The single-particle matrix elements possess the following properties:

$$<j|\Gamma(E\lambda)|j'> = (-1)^{j-j'-\lambda} <j'|\Gamma(E\lambda)|j>,$$

$$<j|\Gamma(M\lambda)|j'> = (-1)^{j+j'-\lambda} <j'|\Gamma(M\lambda)|j>.$$

The matrix element of the $E\lambda$-transition from the high excited to the ground state is

$$M(E\lambda; I^{\pi} \to 0^{+}) = - \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_{t} b^{2t}_{IM}(j_1 m_1, j_2 m_2)\, u^{(+)}_{j_1 j_2} \times$$

$$\times <j_2|\Gamma(E\lambda)|j_1>(j_1 m_1 j_2 m_2|\lambda - \mu) \frac{(-1)^{j_1+j_2-\mu}}{\sqrt{2\lambda+1}}, \tag{26}$$

the functions u_j, v_j are related to the ground state. The matrix element of the $M\lambda$-transition includes $u^{(-)}_{j_1 j_2}$ instead of $u^{(+)}_{j_1 j_2}$. From eq. (26) it is seen that the gamma transitions to the ground state proceed from the two-quasiparticle components of the wave function (8) of the type particle-hole.

The matrix element of the $E\lambda$-transition from the high excited to the one-phonon state described with the wave function (18) is of the form

$$M(E\lambda; I^{\pi} \to \lambda_0\mu_0) = \frac{1}{\sqrt{2\lambda+1}} \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_{t} b_{IM}^{2t}(j_1 m_1, j_2 m_2) \times$$

$$\times \sum_{j,m} \{ <j_1|\Gamma(E\lambda)|j> v^{(-)}_{jj_1} \psi^{\lambda_0 1}_{jj_2} (-1)^{j-m_1-\lambda} (j\, m\, j_1 - m_1|\lambda_0\mu_0) \times$$

$$\times (j\, m\, j_1 - m_1|\lambda\mu) + \dots \} + \frac{1}{\sqrt{2\lambda+1}} \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_{t,i} b_{IM}^{2t,\Omega_i(t)} (j_1 m_1, j_2 m_2) \times$$

$$\times \sum_{j,m} \{ <j|\Gamma(E\lambda)|j_1> u^{(+)}_{j_1 j} \psi^{\lambda_0 1}_{j_2 j} \frac{(-1)^{j_1-m-\mu}}{\sqrt{2j+1}} X^i(j)(j_1 m_1\, j - m|\lambda-\mu) \times$$

$$\times (j_2 m_2\, j\, m|\lambda_0\mu_0) + \dots \} + \frac{1}{\sqrt{2\lambda+1}} \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' \sum_{i} b_{IM}^{2n,\Omega_i(p)} (j_1 m_1, j_2 m_2) \times$$

$$\times \psi^{\lambda_0 1}_{j_1 j_2} (j_1 m_1\, j_2 m_2|\lambda_0\mu_0) \cdot \frac{1}{2} \sum_{j,m} <j|\Gamma(E\lambda)|j> u_{jj'} \frac{(-1)^{j-m}}{\sqrt{2j+1}} X^i(j) \times \tag{27}$$

$$\times (j\, m\, j - m|\lambda\mu) + \frac{1}{\sqrt{2\lambda+1}} \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}}{}' b_{IM}^{2n,2p} (j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) \times$$

$$\times \psi^{\lambda_0 1}_{j_1 j_2} (j_1 m_1\, j_2 m_2|\lambda_0\mu_0) u^{(+)}_{j_3 j_4} <j_4|\Gamma(E\lambda)|j_3> (-1)^{j_3+j_4-\mu} (j_3 m_3 j_4 m_4|\lambda-\mu) +$$

$$+ \frac{-1}{\sqrt{2\lambda+1}} \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}}{}' \sum_{t} b^{4t}_{IM}(j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) \{ <j_2|\Gamma(E\lambda)|j_1> u^{(+)}_{j_1 j_2} \times$$

$$\times (j_1 m_1 j_2 m_2|\lambda-\mu)(-1)^{j_1+j_2-\mu} \psi^{\lambda_0 1}_{j_3 j_4} (j_3 m_3, j_4 m_4|\lambda_0\mu_0) + \dots \} .$$

From the comparison of eqs. (26) and (27) it is seen that the gamma transitions to one-phonon states involve a far larger number of components of the wave function (8) compared with the transition to the ground state. So, side by side with the components particle-hole, the components particle-particle and hole-hole take part in the transtion . The four quasiparticle components and the components two quasiparticles plus phonon Ω_i^+ give a definite contribution.

The matrix element of the $E\lambda$-transition from the high excited to the two-quasiparticle neutron state described by the wave function (20) is of the form:

$$M(E\lambda; I^{\pi} \to I_f (j_f, j_f')) = \frac{1}{\sqrt{2\lambda+1}} \sum_{m_f, m_f'} (j_f m_f j_f' m_f' \mid I_f M_f) \times$$

$$\times \Big\{ \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' b^{2n}_{IM}(j_1 m_1, j_2 m_2) \langle j_f | \Gamma(E\lambda) | j_1 \rangle v^{(-)}_{j_f j_1} \delta_{j_f' j_2} \delta_{m_f' m_2} \times$$

$$\times (-1)^{j_f + m_f + \lambda} (j_f m_f j_1 - m_1 | \lambda\mu) + \dots + \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' b^{2n, \Omega(n)}_{IM}(j_1 m_1, j_2 m_2) \times \qquad (28)$$

$$\times \langle j_f | \Gamma(E\lambda) | j_1 \rangle u^{(+)}_{j_1 j_f} \delta_{j_f' j_2} \delta_{m_f' m_2} (-1)^{j_1 - m_f + \mu} X^i(j_f) \times$$

$$\times (j_1 m_1 j_f - m_f | \lambda - \mu) + \dots + \sum_{\substack{j_1, j_2 \\ m_1, m_2}}{}' b^{2n, \Omega(p)}_{IM} (j_1 m_1, j_2 m_2) \times$$

$$\times (\delta_{j_1 j_f} \delta_{m_1 m_f} \delta_{j_2 j_f'} \delta_{m_2 m_f'} - \delta_{j_1 j_f'} \delta_{m_1 m_f'} \delta_{j_2 j_f} \delta_{m_2 m_f}) \frac{1}{2} \sum_{j,m} < j \| \Gamma (E\lambda) | j > \times$$

$$\times u^{(+)}_{jj'} \frac{(-1)^{j-m}}{\sqrt{2j+1}} X^{i}(j)(j\, m\, j\, -m | \lambda \mu) +$$

$$+ \sum_{\substack{j_1, j_2, j_3, j_4 \\ m_1, m_2, m_3, m_4}} \sum_{t'} b^{2n, 2t'}_{IM} (j_1 m_1, j_2 m_2, j_3 m_3, j_4 m_4) \delta_{j_f j_3} \delta_{m_f m_3} \delta_{j_f' j_4} \delta_{m_f' m_4} \times \tag{28}$$

$$\times < j_2 | \Gamma (E\lambda) | j_1 > u^{(+)}_{j_1 j_2} (j_1 m_1\, j_2 m_2 | \lambda - \mu) \times$$

$$\times (-1)^{j_1 + j_2 - \mu + 1} + \dots \} .$$

The formulas for the $M\lambda$ -transitions can be obtained from (27) and (28) by replacing the single-particle matrix elements and the functions $u^{(+)}_{jj'}$, $v^{(-)}_{jj'}$ by the functions $u^{(-)}_{jj'}$, $v^{(+)}_{jj'}$.

From eqs. (28) it is seen that the gamma transition proceeds only from that two-quasiparticle component of the wave function (8) for which the location of one quasiparticle does not change in the process of transition. Only those four-quasiparticle components of the wave function (8) contribute to the gamma transition to the two-quasiparticle state for which the location of two quasiparticles is the same as in the final state and the two other quasiparticles are particle-hole.

6. The aim of the present paper is not calculate the coefficients b_{IM} of the wave function (8) but to clarify whether it is possible to obtain information on the b_{IM} -values from the experimental data on the reduced probabilities of gamma and alpha decays of the high excited states. The calculations performed have shown that a very restricted number of the components of the wave functions of high excited states are involved in the alpha and gamma transitions. Therefore the analysis of the experimental data on these transitions may give evidence on the magnitude of some components of the high excited state wave function.

We summarize the information on the structure of the high excited states which can be obtained in studying the reduced probabilities of alpha and gamma transitions from the resonances which are manifested in the $(n\alpha)$ and $(n\gamma)$ reactions.

1. The alpha decay to the ground state proceeds from the following components of the wave function (8) : two-quasiparticle, four-quasiparticles $2n2p$ and two quasiparticles plus phonon $\Omega_i^{(+)}$. All the quasiparticle operators must be of the particle type. From the study of the alpha transitions to the ground states it is possible to evaluate the integral contribution of the above components to the normalization condition (10). The reduced probability of the alpha transition from the two-quasiparticle components of the type particle-particle is essentially enhanced. If the integral contribution of the component particle-particle in (8) fluctuates from resonance to resonance then the corresponding alpha-widths should fluctuate as well.

2. The gamma transition to the ground state proceeds from the two-quasiparticle components of the particle-hole type. Since the alpha transition involve the components particle-particle then there are no correlations between the alpha and gamma transitions to the

ground states from a given resonance. The fluctuations of the reduced probabilities of gamma transitions to the ground states in passing from one resonance to another give evidence of the fluctuations of the magnitude of the two-quasiparticle components of the particle-hole type.

3. The alpha and gamma transitions to the one-phonon states involve an essentially larger number of the components of the wave function (8) compared with the transitions to the ground states. Therefore the reduced probabilities of the transitions to the one-phonon states must, on the average, be larger than those for the transitions to the ground states. If the reduced probability of the alpha or gamma transition to the ground state is larger than to the one-phonon one then in this case the wave function (8) contains large two-quasiparticle components of the particle-particle or particle-hole type.

4. The alpha and gamma transitions from the high excited to the two-quasiparticle states involve a considerable number of the components of the wave function (8). However, the choice of these components is restricted by the conditions requiring that one or two-quasiparticles in the high excited state be in the same subshells that the quasiparticles in the final states. The fulfillments of these requirements is unlikely. Therefore it should be expected that the reduced probabilities of the alpha and gamma transitions to the two-quasiparticle states will be smaller than to the one-phonon ones, and possible, to the ground ones.

It is not difficult to make a generalization of the formulas obtained to the cases of even-even deformed nuclei as well as of odd- and odd-odd nuclei. It is possible to obtain the expression for the matrix elements of the transitions to the two-phonon states. In these cases the number of the components of the wave function (8) involved in the transitions essentially increases.

It should be noted that the obtained expressions for the matrix elements of the alpha and gamma transitions are approximate. This is due, first of all, to the simple form of the considered wave functions of the initial and final states (since the wave functions are of the form: the quasiparticle and phonon operators act on the quasiparticle vacuum), secondly, to the treatment of tha alpha decay as a one-stage process. Therfore, some deflections from the quasiparticle selection rules revealed in the matrix elements of the alpha and gamma transitions are possible.

It is interesting to analyse the available experimental data on the basis of the suggested here semimicroscopic description of the structure of the high excited states. It is especially important to systematize the hindrance factors for tha alpha and gamma transitions from the resonances for the estimation of the integral contribution of the definite components of the wave functions of high excited states.

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Reference

1. V.G. Soloviev. Nuclear Structure, Dubna Symp. 1968, p.101, IAEA, 1968.
2. Ю.П. Попов, М. Стэмпински. Письма ЖЭТФ, 7, 126 (1968).
   И. Квитек, Ю.П. Попов. Сообщение ОИЯИ, Р3-4982 (1970).
   Ю.П. Попов, М. Пшитула, К.Г. Родионов, Р.Ф. Руми, М. Стэмпински, В.И. Фурман. Сообщение ОИЯИ, Р3-5073 (1970).
3. R.D. Macfarlane, I. Almodovar. Phys.Rev., 127, 1665 (1962).
   В.Н. Андреев, С.М. Сироткин. ЯФ, 1, 252 (1965).
   K. Beg, R.D. Macfarlane. Nucl.Phys., A129, 571 (1969).
   K. Okatomo. Nucl.Phys., A141, 193 (1970).

4. В.Г. Соловьев. Влияние парных корреляций сверхпроводящего типа на свойствах атомных ядер, Госатомиздат, 1963.
S. Yoshida. Phys.Rev., 123, 2122 (1961).
5. V.G. Soloviev. Nucl.Phys., 69, 1 (1965).
D.R. Bés, R.A. Broglia. Nucl.Phys., 80, 289 (1966).
6. V.G. Soloviev. Phys.Lett., 1, 202 (1962).
H.J. Mang, J.O. Rasmussen. Mat.Fys.Skr.Dan.Vid.Selsk., 2, N3 (1962).
7. S. Yoshida. Nucl.Phys., 38, 380 (1962).
V.G. Soloviev. Atomic Energy Review, 3, N2, 117 (1965).

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