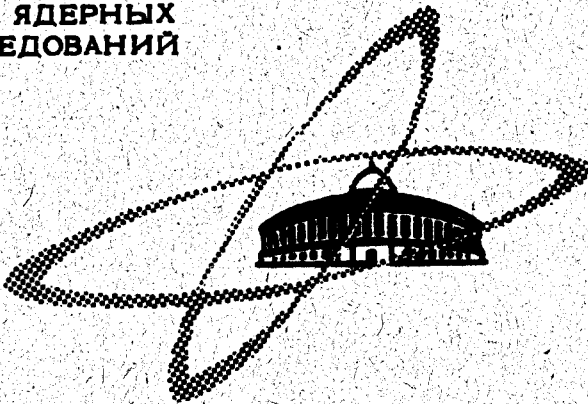


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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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DERIVATION
OF THE KINETIC EQUATIONS
FOR THE SYSTEM WEAKLY COUPLED
TO A THERMAL BATH

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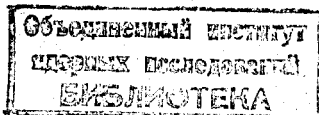
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Submitted to
"Теоретич. и матем. физика"



1. Introduction

In this paper we investigate a relaxation processes in two weakly interacting subsystems, one of which is in the nonequilibrium state and the other is considered as a thermal bath. We are interested in the problem of derivation of the kinetic equations for a certain set of the average values which characterize the nonequilibrium state of the system. As is known, the generalized kinetic equations, i.e. the equations for a certain set of the averages (e.g. for the occupation numbers, spins and so on), describing the nonequilibrium state of the system with small interaction have been obtained by S.V. Peletminski and A.A. Yatsenko^{/1/} and L.A. Pokrovski^{/2/}. The problem of the system weakly coupled to the thermal bath similar to that studied here has been considered in the paper of L.A. Pokrovski^{/3/}. It was assumed there that an external time-dependent field acts on the non-equilibrium system.

In the derivation of the kinetic equations we use the method of the nonequilibrium statistical operator developed by D.N. Zubarev^{/4-5/}. In the following section we consider the construction of

the nonequilibrium statistical operator and derive the kinetic equation for the system weakly coupled to a thermal bath.

It is shown, that the expression for the "collision term" has the same form as in the generalized kinetic equations in papers^{1,2/} but differs by that the states of the thermal bath are taken into account in it, in the averaging. In Section 3, on the basis of the derived equations, in some approximation, the equations similar in structure to the Redfield's type equations for the spin density matrix are obtained. The master equation has been obtained for a particular case. As an example, in Section 4 we consider the longitudinal nuclear spin-lattice relaxation problem and derive the Gorter's relation.

It should be noted that Redfield's type equations have been considered in paper^{3/} using the method of the nonequilibrium statistical operator when on the nonequilibrium system acts an external time-dependent field.

2. Construction of the Nonequilibrium Statistical Operator and Derivation of the Kinetic Equations

Let us consider the relaxation of a small subsystem weakly interacting with a thermal bath. The Hamiltonian of the total system is taken in the following form

$$H = H_1 + H_2 + V, \quad (1)$$

where

$$H_1 = \sum_a E_a a_a^+ a_a$$

and

$$V = \sum_{\alpha, \beta} \Phi_{\alpha\beta} a_{\alpha}^+ a_{\beta}, \quad \Phi_{\alpha\beta} = \Phi_{\beta\alpha}^+.$$

Here \mathcal{H}_1 is the Hamiltonian of the small subsystem, and a_α^+ , a_α are the creation and annihilation operators of quasiparticles in the small subsystem with energies E_α , V is the Hamiltonian of the interaction between the small subsystem and the thermal bath, \mathcal{H}_2 is the Hamiltonian of the thermal bath, which we do not write explicitly. The quantities $\Phi_{\alpha\beta}$ are the operators acting on the thermal bath variables. We are interested in the kinetic stage of the nonequilibrium process in the system weakly coupled to a thermal bath. Therefore, we assume that the state of this system is determined completely by the set of averages $\langle P_{\alpha\beta} \rangle = \langle a_\alpha^+ a_\beta \rangle$ and the state of the thermal bath by $\langle \mathcal{H}_2 \rangle$, where $\langle \dots \rangle$ denotes the statistical average with the nonequilibrium statistical operator, which will be defined below.

We shall follow the method of the nonequilibrium statistical operator^{4,5}. Let us introduce the quasiequilibrium distribution

$$\rho_\alpha(t) = e^{-S(t,0)}, \quad (2)$$

where $S(t,0) = \Omega(t) + \sum_{\alpha,\beta} P_{\alpha\beta}(0) F_{\alpha\beta}(t) + \beta \mathcal{H}_2(0)$

is the operator of the entropy and $\Omega = \ln \text{Sp} e^{-\sum_{\alpha,\beta} P_{\alpha\beta} F_{\alpha\beta}(t) - \beta \mathcal{H}_2}$.

Here $F_{\alpha\beta}(t)$ are the thermodynamic parameters conjugated with $\langle P_{\alpha\beta} \rangle$, β is the reciprocal temperature of the thermal bath. All the operators are considered in the Heisenberg representation.

The nonequilibrium statistical operator of the total system is constructed in the following way

$$\rho(t) = e^{-S(t,0)}, \quad (3)$$

where

$$\tilde{S}(t,0) = \epsilon \int_{-\infty}^0 dt_1 e^{\epsilon t_1} (\Omega(t+t_1) + \sum_{\alpha\beta} P_{\alpha\beta}(t_1) F_{\alpha\beta}(t+t_1) + \beta \mathcal{H}_2(t_1))$$

is the "quasi-invariant" part of the operator of the entropy and ϵ is the infinitesimal quantity, which we let tend to zero, but only after the thermodynamic limit has been taken. The parameters $F_{\alpha\beta}(t)$ are determined from the conditions^{/4-5/}

$$\langle P_{\alpha\beta} \rangle = \langle P_{\alpha\beta} \rangle_q, \quad (4)$$

where $\langle \dots \rangle_q$ is the statistical averaging with the quasiequilibrium statistical operator ρ_q . The fulfilment of the conditions (4) guarantees the conservation of the normalization after taking the invariant part.

In the derivation of the kinetic equations we shall use the perturbation theory in a "smallness interaction" and shall assume that $\langle \Phi_{\alpha\beta} \rangle_q = 0$ while the other terms can be added to the renormalized energy of the subsystem.

The nonequilibrium statistical operator (3) can be rewritten as

$$\rho(t) = Q^{-1} e^{-L(t)}, \quad (5)$$

where

$$L(t) = \epsilon \int_{-\infty}^0 dt_1 e^{\epsilon t_1} \left\{ \sum_{\alpha,\beta} P_{\alpha\beta}(t_1) F_{\alpha\beta}(t+t_1) + \beta \mathcal{H}_2(t_1) \right\} \quad (6)$$

and Q is the normalizing factor.

Integrating in Eq. (6) by parts, we obtain

$$L(t) = \sum_{\alpha, \beta} P_{\alpha\beta} F_{\alpha\beta}(t) + \beta \mathcal{H}_2 - \int_{-\infty}^0 dt_1 e^{\epsilon t_1} \left\{ \sum_{\alpha, \beta} \dot{P}_{\alpha\beta}(t_1) F_{\alpha\beta}(t+t_1) + \sum_{\alpha, \beta} P_{\alpha\beta}(t_1) \frac{\partial F_{\alpha\beta}(t+t_1)}{\partial t_1} + \beta \dot{\mathcal{H}}_2(t_1) \right\}. \quad (7)$$

For further considerations it is convenient to write ρ_q as

$$\rho_q = \rho_1 \cdot \rho_2 = Q_q^{-1} e^{-A}, \quad (8)$$

where $\rho_1 = Q_1^{-1} \exp \left\{ -\sum_{\alpha, \beta} P_{\alpha\beta} F_{\alpha\beta}(t) \right\}$; $Q_{s_1} = \text{Sp} \exp \left\{ -\sum_{\alpha, \beta} P_{\alpha\beta} F_{\alpha\beta}(t) \right\}$,

$$\rho_2 = Q_2^{-1} \exp \left\{ -\beta \mathcal{H}_2 \right\}; \quad Q_2 = \text{Sp} \exp \left\{ -\beta \mathcal{H}_2 \right\},$$

$$Q_q = Q_1 \cdot Q_2, \quad A = \sum_{\alpha, \beta} P_{\alpha\beta} F_{\alpha\beta}(t) + \beta \mathcal{H}_2.$$

We start from the kinetic equations for $\langle P_{\alpha\beta} \rangle$ in the implicit form

$$\frac{d\langle P_{\alpha\beta} \rangle}{dt} = -i \langle [P_{\alpha\beta}, \mathcal{H}] \rangle = -i (E_{\beta} - E_{\alpha}) \langle P_{\alpha\beta} \rangle - i \langle [P_{\alpha\beta}, V] \rangle. \quad (9)$$

and restrict ourselves to the second-order in interaction in calculating the right-hand side of Eq. (9). To this end we must obtain

$\rho(t)$ in the first-order in V . Taking into account these remarks we calculate

$$\begin{aligned}
\frac{\partial F_{\alpha\beta}(t+t_1)}{\partial t_1} &= i(E_\beta - E_\alpha) F_{\alpha\beta}(t+t_1) - i \sum_{\alpha_1, \beta_1} \frac{\partial F_{\alpha\beta}(t+t_1)}{\partial \langle P_{\alpha_1 \beta_1} \rangle} \times \\
&\times \langle [P_{\alpha_1 \beta_1}(t_1), V(t_1)] \rangle = i(E_\beta - E_\alpha) F_{\alpha\beta}(t+t_1) - i \sum_{\substack{\alpha_1, \beta_1 \\ \alpha_2}} \frac{\partial F_{\alpha\beta}(t+t_1)}{\partial \langle P_{\alpha_1 \beta_1} \rangle} \times \\
&\times (\langle \Phi_{\beta_1 \alpha_2} P_{\alpha_2 \alpha_2} \rangle - \langle \Phi_{\alpha_2 \alpha_1} P_{\alpha_2 \beta_1} \rangle).
\end{aligned} \tag{10}$$

Restricting ourselves to the linear terms in Eq. (10), we obtain

$$\begin{aligned}
\frac{\partial F_{\alpha\beta}(t+t_1)}{\partial t_1} &\approx i(E_\beta - E_\alpha) F_{\alpha\beta}(t+t_1) - i \sum_{\substack{\alpha_1, \beta_1 \\ \alpha_2}} \frac{\partial F_{\alpha\beta}(t+t_1)}{\partial \langle P_{\alpha_1 \beta_1} \rangle} \times \\
&\times \{ \langle \Phi_{\alpha_1 \alpha_2} \rangle \langle P_{\alpha_1 \alpha_2} \rangle - \langle \Phi_{\alpha_2 \alpha_1} \rangle \langle P_{\alpha_2 \alpha_1} \rangle \} = i(E_\beta - E_\alpha) F_{\alpha\beta}(t+t_1).
\end{aligned} \tag{11}$$

The quantities, $\dot{P}_{\alpha\beta}(t_1)$ and $\dot{H}_2(t_1)$ in the first-order in interaction have the form

$$\begin{aligned}
\dot{P}_{\alpha\beta}(t_1) &= -i(E_\beta - E_\alpha) P_{\alpha\beta}(t_1) - i [P_{\alpha\beta}(t_1), V(t_1)] \\
\dot{H}_2(t_1) &= -i [H_2(t_1), V(t_1)].
\end{aligned} \tag{12}$$

Here and below all the operators are taken in the interaction representation.

Using Eqs. (11) and (12) we find

$$L(t) = A + i \int_{-\infty}^0 dt_1 e^{\epsilon t_1} \left[\sum_{\alpha\beta} P_{\alpha\beta}(t_1) F_{\alpha\beta}(t+t_1) + \beta H_2(t_1), V(t_1) \right]. \tag{13}$$

Following [3] it is easy to check that the expression $\sum_{\alpha, \beta} P_{\alpha\beta}(t_1) F_{\alpha\beta}(t+t_1) + \beta H_2$ is independent of t_1 in the zero-order in interaction and consequently is equal to A . Then for $\rho(t)$ in the linear approximation in interaction V we have

$$\rho(t) = \rho_q - i \rho_q \int_{-\infty}^0 dt_1 e^{\epsilon t_1} \int_0^1 d\lambda e^{\lambda A} [A, V(t_1)] e^{-\lambda A}. \quad (14)$$

Now, using the relation [3]

$$e^{\lambda A} [A, V(t_1)] e^{-\lambda A} = \frac{d}{d\lambda} e^{\lambda A} V(t_1) e^{-\lambda A}.$$

and integrating in Eq. (14) over the λ one can easily show that

$$\rho(t) = \rho_q - i \int_{-\infty}^0 dt_1 e^{\epsilon t_1} [V(t_1), \rho_q]. \quad (15)$$

Finally, with the aid of Eq. (15) we obtain the kinetic equations for $\langle P_{\alpha\beta} \rangle$ in the form

$$\frac{d\langle P_{\alpha\beta} \rangle}{dt} = -i(E_\beta - E_\alpha) \langle P_{\alpha\beta} \rangle - \int_{-\infty}^0 dt_1 e^{\epsilon t_1} \langle [P_{\alpha\beta}, V], V(t_1) \rangle_q. \quad (16)$$

The last term of the right-hand side of Eq. (16) can be called the generalized "collision integral". Thus we see that the collision term for the system weakly coupled to the thermal bath has a convenient form of the double commutator as in the generalized kinetic equations [1, 2] for the system with small interaction.

It should be emphasized, that the suggestion about the model form of the Hamiltonian is nonessential. We shall start again from the Hamiltonian given by Eq. (1), in which we do not specify already the form of H_1 and V . We assume that the state of the

nonequilibrium system is characterized completely by some set of the average values $\langle P_k \rangle$ and the state of the thermal bath by $\langle H_2 \rangle$. We confine ourselves by such systems for which $[H_1, P_k] = \sum_{\ell} a_{k\ell} P_{\ell}$, where $a_{k\ell}$ is some c-numbers. Then we assume that $\langle V \rangle_q \approx 0$, where $\langle \dots \rangle_q$ denotes the statistical average with the quasiequilibrium statistical operator

$$\rho_q = Q_q^{-1} \exp \{ - \sum_k P_k F_k(t) - \beta H_2 \}$$

and $F_k(t)$ are the parameters conjugated with $\langle P_k \rangle$. Following the method used above in the derivation of the equations (16), we can obtain the generalized kinetic equations for $\langle P_k \rangle$ with an accuracy of the terms which are quadratic in interaction, in the form

$$\frac{d\langle P_k \rangle}{dt} = i \sum_{\ell} a_{k\ell} \langle P_{\ell} \rangle - \int_{-\infty}^0 e^{\epsilon t_1} \langle [[P_k, V], V(t_1)] \rangle_q dt_1. \quad (16a)$$

In conclusion, we remark that if one constructs the nonequilibrium statistical operator putting $H_2 \rightarrow H_2 + V$, which turns to the Gibbsian canonical distribution in the limit, then one can obtain also the equations (16) but in a more complicated way.

3. Obtaining of the Master Equation and the Redfield's Type Equations

In the previous section we have obtained kinetic equations for $\langle P_{\alpha\beta} \rangle$ in a general form. Our next task is to write the equations (16) in an explicit form. We remark that $V(t_1) = \sum_{\alpha, \beta} \Phi_{\alpha\beta}(t_1) a_{\alpha}^{\dagger} a_{\beta}$; where

$$\tilde{\Phi}_{\alpha\beta}(t_1) = e^{iH_2 t_1} \Phi_{\alpha\beta} e^{-iH_2 t_1} e^{i(E_\alpha - E_\beta)t_1}$$

Now, we calculate the double commutator in the right-hand side of Eq. (16)

$$\begin{aligned} \langle [[P_{\alpha\beta}, V], V(t_1)] \rangle_q &= \sum_{\alpha_1 \beta_1} \{ \langle \Phi_{\beta\alpha_1} \tilde{\Phi}_{\alpha_1 \beta_1}(t_1) \rangle_q \langle P_{\alpha\beta} \rangle_q \\ &+ \langle \tilde{\Phi}_{\beta_1 \alpha_1}(t_1) \Phi_{\alpha_1 \alpha} \rangle_q \langle P_{\beta_1 \beta} \rangle_q - \langle \Phi_{\alpha_1 \alpha} \tilde{\Phi}_{\beta\beta_1}(t_1) \rangle_q + \langle \tilde{\Phi}_{\alpha\alpha}(t_1) \Phi_{\beta\beta_1} \rangle_q \times \\ &\times \langle P_{\alpha_1 \beta_1} \rangle \}, \end{aligned} \quad (17)$$

where we restricted ourselves to the linear terms in the concentration of the quasiparticles.

The correlation functions $\langle \Phi_{\alpha_1 \beta_1} \tilde{\Phi}_{\alpha_2 \beta_2}(t) \rangle_q$ and $\langle \tilde{\Phi}_{\alpha_1 \beta_1}(t) \Phi_{\alpha_2 \beta_2} \rangle_q$ are connected with its spectral intensities in the following way

$$\langle \Phi_{\alpha_1 \beta_1} \tilde{\Phi}_{\alpha_2 \beta_2}(t) \rangle_q = \int_{-\infty}^{\infty} J_{\alpha_2 \beta_2, \alpha_1 \beta_1}(\omega) e^{-i(\omega - E_{\alpha_2} - E_{\beta_2})t} d\omega \quad (18)$$

$$\langle \tilde{\Phi}_{\alpha_1 \beta_1}(t) \Phi_{\alpha_2 \beta_2} \rangle_q = \int_{-\infty}^{\infty} J_{\alpha_2 \beta_2, \alpha_1 \beta_1}(\omega) e^{i(\omega + E_{\alpha_1} - E_{\beta_1})t} d\omega$$

Substituting Eqs. (17) and (18) into Eq. (16) and taking into account the notations

$$\sum_{\alpha_1} \int_{-\infty}^{\infty} \langle \Phi_{\beta\alpha_1} \tilde{\Phi}_{\alpha_1 \beta_1}(t_1) \rangle_q e^{\epsilon t_1} dt_1 = i \sum_{\alpha_1} \int_{-\infty}^{\infty} d\omega \frac{J_{\alpha_1 \beta_1, \beta\alpha_1}(\omega)}{\omega - E_{\alpha_1} + E_{\beta_1} + i\epsilon} = K_{\beta\beta_1}$$

and

$$\int_{-\infty}^0 \{ \langle \Phi_{a_1 a} \tilde{\Phi}_{\beta \beta_1}(t_1) \rangle + \langle \tilde{\Phi}_{a_1 a}(t_1) \Phi_{\beta \beta_1} \rangle \} e^{\epsilon t_1} dt_1 =$$

$$= i \int_{-\infty}^{\infty} d\omega J_{\beta \beta_1, a_1 a}(\omega) \left\{ \frac{1}{\omega - E_{\beta} + E_{\beta_1} + i\epsilon} - \frac{1}{\omega - E_{a_1} + E_{\beta_1} - i\epsilon} \right\} = K_{a\beta, a_1 \beta_1},$$

one can rewrite the kinetic equations for $\langle P_{a\beta} \rangle$ as

$$\frac{d\langle P_{a\beta} \rangle}{dt} = i(E_a - E_{\beta}) \langle P_{a\beta} \rangle - \sum_{\beta_1} \{ K_{\beta \beta_1} \langle P_{a\beta_1} \rangle + K_{a\beta_1}^* \langle P_{\beta_1 \beta} \rangle \} +$$

$$+ \sum_{a_1 \beta_1} K_{a\beta, a_1 \beta_1} \langle P_{a_1 \beta_1} \rangle. \quad (19)$$

The above result is similar in structure to the Redfield equation for the spin density matrix^[7] when the external time-dependent field is absent.

If one can confine itself by the diagonal averages $\langle P_{aa} \rangle$ only, the last equation may be transformed to give

$$\frac{d\langle P_{aa} \rangle}{dt} = \sum_{\beta} K_{a\omega \beta \beta} \langle P_{\beta \beta} \rangle - (K_{aa} + K_{aa}^*) \langle P_{aa} \rangle,$$

where

$$K_{aa, \beta \beta} = 2\pi J_{a\beta, \beta a} (E_a - E_{\beta}) = W_{\beta \rightarrow a}$$

and

$$K_{aa} + K_{aa}^* = 2\pi \sum_{\beta} J_{\beta a, a\beta} (E_{\beta} - E_a) = \sum_{\beta} W_{a \rightarrow \beta}.$$

Here $W_{\beta \rightarrow a}$ and $W_{a \rightarrow \beta}$ are the transition probabilities expressed in the spectral intensity terms. Using the properties of the spectral intensities, it is easy to verify that the transition probabilities satisfy the relation of the detailed balance

$$\frac{W_{\beta \rightarrow a}}{W_{a \rightarrow \beta}} = \frac{e^{-\beta E_a}}{e^{-\beta E_\beta}}.$$

Finally, we have

$$\frac{d\langle P_{aa} \rangle}{dt} = \sum_{\beta} W_{\beta \rightarrow a} \langle P_{\beta\beta} \rangle - \sum_{\beta} W_{a \rightarrow \beta} \langle P_{aa} \rangle. \quad (20)$$

This equation has the usual form of the master equation.

4. Longitudinal Nuclear Spin-Lattice Relaxation

As an example in this section we consider the problem of spin-lattice relaxation.

Let us consider the behaviour of a spin system with the Hamiltonian \mathcal{H}_S weakly coupled by a time-independent perturbation V to a temperature reservoir or a crystal lattice with the Hamiltonian \mathcal{H}_L .

The total Hamiltonian has the form

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_L + V, \quad (21)$$

where $\mathcal{H}_S = -a \sum_i I_i^z$, $a = \gamma H_0$ and I_i^z is the operator of the z -component of the spin at the site i , H_0 is the time-independent external field applied in the z -direction, γ is the gyromagnetic coefficient.

Now we introduce $a_{i\lambda}^+$ and $a_{i\lambda}$ the creation and annihilation operators of the spin in the site i with z -component equal to λ , where $-I \leq \lambda \leq I$.

Then we have

$$I_1^z = \sum_{\lambda} \lambda a_{1\lambda}^+ a_{1\lambda} = \sum_{\lambda} \lambda n_{1\lambda},$$

and consequently $\mathcal{H}_s = \sum_{\lambda, i} E_{\lambda} n_{i\lambda}$, $E_{\lambda} = -a_{\lambda}$. Following section 2 we write the Hamiltonian of the interaction as

$$V = \sum_{i\mu, \lambda} \sum_{i\lambda, i\mu} \Phi_{i\lambda, i\mu} a_{i\lambda}^+ a_{i\mu}, \quad \Phi_{i\lambda, i\mu} = \Phi_{i\mu, i\lambda}^+.$$

Here $\Phi_{i\lambda, i\mu}$ are the operators acting only on the lattice variables.

In agreement with Eq. (8) we construct the quasiequilibrium statistical operator

$$\rho_q = \rho_L \cdot \rho_S, \quad (22)$$

where

$$\rho_L = Q_L^{-1} \exp \{ -\beta \mathcal{H}_L \}, \quad Q_L = \text{Sp} \exp \{ -\beta \mathcal{H}_L \}$$

and

$$\rho_S = Q_S^{-N} \exp \{ -\beta_S(t) \mathcal{H}_S \}, \quad Q_S = \frac{\text{sh} \frac{\beta_S(t)}{2} a(2I+1)}{\text{sh} \frac{\beta_S(t)}{2} a}.$$

Here β_S is the reciprocal spin temperature and N is the total number of spins in the system.

The average values $\langle n_{i\lambda} \rangle = \langle a_{i\lambda}^+ a_{i\lambda} \rangle$ correspond to the equation (see Eq. (20))

$$\frac{d\langle n_{i\lambda} \rangle}{dt} = \sum_{\mu} W_{\mu \rightarrow \lambda} (ii) \langle n_{i\mu} \rangle - \sum_{\mu} W_{\lambda \rightarrow \mu} (ii) \langle n_{i\lambda} \rangle, \quad (23)$$

where

$$W_{\lambda \rightarrow \mu} (ii) = 2\pi J_{\Phi_{\mu i, \lambda i}} \Phi_{\lambda i, \mu i} (E_{\mu} - E_{\lambda})$$

and

$$W_{\mu \rightarrow \lambda} (ii) = 2\pi J_{\Phi_{\lambda i, \mu i}} \Phi_{\mu i, \lambda i} (E_{\lambda} - E_{\mu}).$$

We remark that $\langle n_{i\lambda} \rangle = \langle n_{\lambda} \rangle = Q_S^{-1} e^{-\beta_S E_{\lambda}}$. Then we have

$$\frac{d\langle n_{\lambda} \rangle}{dt} = \sum_{\mu} W_{\mu \rightarrow \lambda} \langle n_{\mu} \rangle - \sum_{\mu} W_{\lambda \rightarrow \mu} \langle n_{\lambda} \rangle.$$

Here

$$W_{\lambda \rightarrow \mu} = \frac{1}{N} \sum_i W_{\lambda \rightarrow \mu} (ii)$$

and

$$W_{\mu \rightarrow \lambda} = \frac{1}{N} \sum_i W_{\mu \rightarrow \lambda} (ii).$$

It is easily seen that $W_{\mu \rightarrow \lambda} = e^{\beta_S (E_{\mu} - E_{\lambda})} W_{\lambda \rightarrow \mu}$. Finally, for β_S we find the equation

$$\frac{d\beta_S}{dt} = \frac{\frac{1}{2} \sum_{\mu, \lambda} (\lambda - \mu) W_{\lambda \rightarrow \mu} [1 - e^{-(\beta - \beta_S)(E_{\lambda} - E_{\mu})}] e^{-\beta_S E_{\lambda}}}{\frac{Q_S}{a} \frac{\partial^2 \ln Q_S}{\partial \beta_S^2}} \quad (24)$$

In the derivation of Eq. (24) we took into account that $\langle I^z \rangle = \sum_{\lambda} \lambda \langle n_{\lambda} \rangle$ and

$$\frac{d\langle I^z \rangle}{dt} = -\frac{1}{a} \frac{d\beta_S}{dt} \frac{\partial^2 \ln Q_S}{\partial \beta_S^2} = -\frac{1}{a} \frac{d\beta_S}{dt} (\langle (I^z)^2 \rangle - \langle I^z \rangle^2).$$

In the high-temperature approximation we obtain

$$\frac{d\beta_s}{dt} = \frac{\beta - \beta_s}{T_1},$$

where T_1 is the longitudinal time of the spin-lattice relaxation

$$\frac{1}{T_1} = \frac{1}{2} \frac{\sum_{\lambda, \mu} (\lambda - \mu)^2 W_{\lambda \rightarrow \mu}}{\sum_{\lambda} \lambda^2}.$$

The above expression is the well-known Gorter relation^[7].

Conclusion

The results of the preceding sections show that the method of the nonequilibrium statistical operator is well applied to the study of the relaxation processes. Our consideration is simple enough and it can be used in a number of concrete problems. We remark that in a similar way it is possible to obtain the Schrödinger type equation for the average amplitudes $\langle a_a \rangle$ and $\langle a_a^+ \rangle$ for the system weakly coupled to the thermal bath. This will be the topic of our subsequent paper.

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