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ON THE DEPENDENCE OF QQ-FORCE STRENGTH ON DEFORMATION

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ON THE DEPENDENCE OF QQ-FORCE STRENGTH ON DEFORMATION

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О зависимости силы QQ - взаимодействия от деформации

Предположен метод теоретической оценки зависимости эффективных двухчастичных взаимодействий в ядре от деформации.

Препринт Объединенного института ядерных исследований.
Дубна, 1969

Bochnacki Z. E4-4843

On the Dependence of QQ - Force Strength on Deformation

QQ - force for RPA calculations with Nilsson's single particle potential is studied and its dependence on nuclear deformation is estimated.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1969

The dependence of the effective two-body forces in the nucleus on the density distribution, which, according to references [4,5], is much stronger for the forces used in RPA calculation than for, say, Brueckner's reaction matrix, is particularly important in deformed nuclei where it can introduce the dependence of the effective force strength on deformation. Such an effect cannot be disregarded in the calculations, especially when their aim is to study the changes in nuclear properties with increasing deformation, as it is the case when the fission processes are considered.

Some evidence of the QQ -force dependence on deformation comes from the analysis of the 2^+ vibrational states properties in even-nuclei with different values of the equilibrium deformation. It is well known, for instance, that one needs lower value of the QQ - force strength κ_{22} to get the 2^+ state energies in Os and W isotopes than in other, more deformed nuclei in the rare earth region. The same tendency is observed for the ground states of $A > 220$ nuclei. Recently the 2^+ level energy was calculated for the deformations corresponding to the fission barrier of some $A > 220$ nuclei [1,2]. In this case the value of κ'_{22} required to account for the observed energies turned out to be larger by about 50% than the average for the ground state deformation.

The evidence mentioned above is however not conclusive enough in view of many approximations used and uncertainties in the choice of parameters in the calculations of this type. It is thus interesting to make a theoretical estimate of the dependence on deformation of the QQ -force corresponding to the single particle potentials used in the RPA calculations, when their density dependence is specified by the generally used self-consistency conditions. We do it here for the Nilsson's single particle potential, for which the calculations of the reference [1] were made.

A method of calculating the forces appropriate for a given single particle potential with the self-consistency conditions known was described in references [4,5]. In the form applied here it may be considered as a simple generalization of the method proposed earlier by A.Bohr and B.M. Mottelson for calculating the multipole force strength [3]. The effective force F for RPA calculations is connected with the single particle potential U by the relation

$$F = \frac{\delta U}{\delta \rho}, \tag{1}$$

where ρ is the density matrix. When we are interested in the changes of ρ of definite symmetry described by a parameter $Q = \int \hat{Q} \rho \, d^3 r$ the derivative in (1) may be rewritten as

$$F_Q = \frac{\partial U}{\partial Q} \frac{\delta Q}{\delta \rho} = \frac{\partial U}{\partial Q} \hat{Q} . \tag{2}$$

To calculate the derivative $\partial U/\partial Q$ the self-consistency condition connecting the symmetry of the density distribution with the symmetry of the potential must be used. We assume here that these symmetries are the same for the volume conserving Nilsson's potential. In order to make calculations to all orders in the deformation parameters we use the volume conserving parameters σ, γ of the reference /6/ ($\sigma \approx 0{,}631\,\beta$) . The self-consistency conditions take then the form

$$Q_{20} = \sqrt{\frac{16\pi}{5}} \int r^2 Y_{20}\, \rho\, d^3r = \frac{1}{3}\left[2e^{2\sigma\cos\gamma} - e^{2\sigma\cos(\gamma-\frac{2\pi}{3})} - e^{2\sigma\cos(\gamma+\frac{2\pi}{3})}\right] \langle r^2 \rangle A \tag{3}$$

$$Q_{22} = \sqrt{\frac{32\pi}{15}} \int r^2 Y_{22}\, \rho\, d^3r = \frac{1}{3}\left[e^{2\sigma\cos(\gamma-\frac{2\pi}{3})} - e^{2\sigma\cos(\gamma+\frac{2\pi}{3})}\right] \langle r^2 \rangle A.$$

The $\hat{Q}_{20}$ and $\hat{Q}_{22}$ parts of the Nilsson's potential, which give rise to the $\hat{Q}_{20}\hat{Q}_{20}$ and $\hat{Q}_{22}\hat{Q}_{2-2}$ components of the effective force respectively, have the form .

$$U_{20} = \frac{1}{12} M \overset{0}{\omega_0}{}^2 \left[2e^{-2\sigma\cos\gamma} - e^{-2\sigma\cos(\gamma-\frac{2\pi}{3})} - e^{-2\sigma\cos(\gamma+\frac{2\pi}{3})}\right] \hat{Q}_{20}$$

$$U_{22} = \frac{1}{8} M \overset{0}{\omega_0}{}^2 \left[e^{-2\sigma\cos(\gamma-\frac{2\pi}{3})} - e^{-2\sigma\cos(\gamma+\frac{2\pi}{3})}\right] \hat{Q}_{22}. \tag{4}$$

The corresponding components of the effective force are

$$F_\beta = \frac{\partial U_{20}}{\partial \sigma}\frac{\partial \sigma}{\partial Q_{20}}\hat{Q}_{20} = -\kappa_{20} r_1^2 r_2^2 Y_{20}(1) Y_{20}(2)$$

$$F_\gamma = \frac{\partial U_{22}}{\partial \gamma}\frac{\partial \gamma}{\partial Q_{2-2}}\hat{Q}_{2-2} = -\kappa_{22} r_1^2 r_2^2 Y_{22}(1) Y_{2-2}(2). \qquad (5)$$

The derivatives can now be easily calculated from (3) and (4). For $\gamma = 0$ one gets:

$$\kappa_{20} = \frac{4\pi}{5}\frac{M\omega_0^{0\,2}}{A\langle r^2\rangle}\frac{2e^{-2\sigma}+e^{\sigma}}{2e^{2\sigma}+e^{-\sigma}} \; .$$

$$\kappa_{22} = \frac{4\pi}{5}\frac{M\omega_0^{0\,2}}{A\langle r^2\rangle}e^{2\sigma} \; . \qquad (6)$$

For $\sigma = 0$ this is exactly the estimate of the QQ-force strength obtained in reference /3/. For $\sigma > 0$, κ_{20} decreases slowly with growing deformation, while κ_{22} increases quite fast. The opposite is true for negative deformations. The σ-dependence given by (6) increases by about 50% the value of κ_{22} when the deformation changes from the equilibrium value for $A > 220$ nuclei to the region of the fission barrier ($\beta \approx 0.5$) . This seems to support the conclusion of the references /1/. The σ-dependence (6) is however too weak to explaine the variation of κ_{22} for the ground state deformation, obtained in the same reference. This may be connected with differences in renormalization of κ_{22} in different nuclei and will need a thorough analysis of the details of the calculation procedure.

A similar technique may be applied for estimating the strength dependence on deformation in case of other components of the effective force and for more realistic single particle potentials.

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