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ON DETERMINATION OF THE PAIRING STRENGTH FROM THE CONDITION FOR THE TOTAL ENERGY MINIMUM

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A deep analogy of pairing phenomena with other modes of nuclear motion was recently discovered [1] and shown to be fruitful (see e.g. references [1-4]). At the level of nuclear Hamiltonian the analogy is best pronounced when the total Hamiltonian is written in the form of a sum of the single particle and pairing fields:

$$H = H_{s.p.} + H_{pairing} = H_{s.p.} - \Delta \sum_{\nu>0} a^{+}_{\nu} a^{+}_{-\nu} - \Delta^{*} \sum a_{-\nu} a_{\nu} . \qquad (1)$$

In the simplest, static case, the minima of the BCS-average value of the Hamiltonian (1) for fixed average number of particles and with self-consistency condition

$$\Delta = G < \sum_{\nu>0} a^{+}_{\nu} a^{+}_{-\nu} >_{BCS} \qquad (2)$$

correspond to the usual solutions for pairing parameters Δ and λ.

It is interesting to go still further and look for a physically sound self-consistency condition which would not involve the pairing force strength G . Such condition would enable us to find the equilibrium value of Δ without using (2). By going back to the relation (2) one would then be able to fix the value of the pairing strength G .

The simplest and probably the most obvious condition is to require that the Hamiltonian (1) describes the nucleus with a fixed radius, no matter how large the pairing field is. This means that changing the pairing part of the Hamiltonian we adjust the single particle field in such a way that the average nuclear density remains constant.

It may be demonstrated on a simple easily solvable model that the condition $\langle r^2 \rangle_{BCS}$ = const. is sufficient to give minimum to the total energy. Let the lowest N shells and half of the N-th shell be filled with particles of one kind, N being the quantum number of the harmonic oscillator. For the sake of simplicity let the pairing affect $(N-1)$, N and $(N+1)$ shells only and assume for them the same degeneracy $2\Omega_N$. One deals then with a symmetric case as far as pairing is concerned and the chemical potential λ is equal to $(N+\frac{3}{2})\hbar\omega$. The total energy for the BCS ground state takes the form

$$E = \hbar\omega \left\{ \frac{3}{4} B + \frac{18}{4} \frac{N+1}{N+2} \Omega_N - \Omega_N \left(\frac{1.5+x^2}{\sqrt{1+x^2}} + \frac{x}{2} \right) \right\}, \tag{3}$$

where

$$B = \sum_{n=0}^{N-1} (n+\frac{3}{2})(n+1)(n+2) + \Omega_N (N+\frac{3}{2}), \qquad x = \frac{\Delta}{\hbar\omega}.$$

The self-consistency condition

$$\langle r^2 \rangle_\Delta \equiv \sum_{\nu>0} r^2_{\nu\nu} \, 2 V^2_\nu (\Delta) = \langle r^2 \rangle_{\Delta=0} \tag{4}$$

means that for each value of Δ the harmonic oscillator potential should be taken with

$$\omega = \omega_0 \{ 1 + \frac{C\Omega_N}{B} (1 - \frac{1}{\sqrt{1+x^2}}) \}, \qquad (5)$$

where $C = \frac{3N(2N+3)+7.5}{(N+1)(N+2)}$, ω_0 is the value of ω for $\Delta = 0$ and where in calculating the radius real degeneracies of the harmonic oscillator were used for all the levels.

For sufficiently large N , when Ω_N / B is small and $C \approx 6$, the energy (3) with ω given by (5) has a minimum at $\Delta_{eq} = 0.13 \hbar\omega$, the position of which does not practically depend on N . For large Δ , E decreases again and one would have to include pairing in $N+2$ and $N-2$ shells to keep it increasing.

Going back to the relation (2) one gets the expression for the pairing force strength G . The effective G for calculations within one shell is

$$G \approx \frac{2\Delta_{eq}}{\Omega_N} . \qquad (6)$$

The variation of G_{eff} from shell to shell may be connected with its A-dependence by taking $\Omega_N \approx 0.6 A^{2/3}$. The equilibrium value of Δ obtained above gives then $G_{eff} \approx 18/A$ MeV which is close to the pairing force strength used in realistic calculations. The same model with the N-th shell completely filled gives, $\Delta_{eq} = 0$. This is again just what one would expect for the model having anything in common with reality.

Quite reasonable results of the simple model make the self-consistency condition (4) interesting from the point of view of more realistic calculations. Such calculations are now in progress.

Some words should probably be said here concerning the general problem of connection of the strength of different field producing components of the effective two-body nuclear force with the parameters of the average single particle potential. It may look strange at first sight that such an explicitly two-body part of nuclear hamiltonian as the pairing force is so intimately connected with the single particle field. It may be shown however (see e.g. reference /5/) that the effective force of quite a general type (at least in the sense that it can be used with reasonable success for nuclei at any place of the periodic system) is determined by two fundamental characteristics of nuclear matter, namely by its density and binding energy per particle. The same characteristics determine the single particle potential in the nucleus. Any component of the two-body force which contributes to the average field with distinct symmetry properties and is to be accounted for by the Hartree-like method can thus be connected in principle with the average field parameters.

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Бохнацки З. E4-4842

Об определении силы парного взаимодействия из условия минимума полной энергии

Предполагается использовать условие постоянного среднего радиуса ядра для определения силы парных взаимодействий методом минимализации полной энергии ядра.

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On Determination of the Pairing Strength from the Condition for the Total Energy Minimum

The use of the condition $\langle r^2 \rangle_{BCS}$ = const. is proposed to determine the pairing force strength by minimalization of the total energy of the nucleus.

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