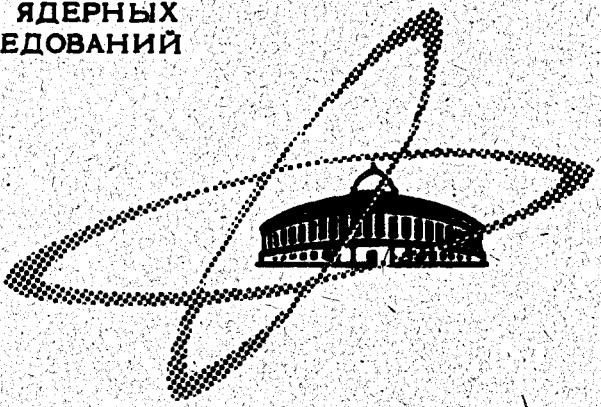
ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна



E4 - 4774

S.I. Gabrakov, A.A. Kuliev, N.I. Pyatov

THE STATES $I^{\pi} = 1^{+}$ IN DEFORMED EVEN-EVEN NUCLEI

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1969

E4 - 4774

S.I. Gabrakov, A.A. Kuliev, N.I. Pyatov

THE STATES $I^{\pi} = 1^{+}$ IN DEFORMED EVEN-EVEN NUCLEI

Submitted to ЯФ.

Introduction

Recent experimental and theoretical investigations as a rule show that the spin-dependent part of the residual interaction in odd-mass nuclei leads to polarization effects, which influence considerably the magnitude of magnetic moments and probabilities of allowed β-decays. Because of these effects the values of the g_s-factors of the nucleons in a nucleus differ noticeably from the corresponding values for free nucleons. In fact, this is equivalent to renormalization of the single-particle matrix elements of the odd nucleon. Such effects are also observed for the matrix elements of β-decay.

The effect of the spin-spin force in atomic nuclei have been studied by various authors by means of perturbation theory[1] and the theory of finite Fermi-systems[2]. Similar calculations have been done by us[3], in the framework of T.D.Approximation. Starting from a simple model, we assumed that in even-even deformed nuclei there exist oscillations of the magnetic dipole moment. These oscillations generate 1^+ states. Then the spin polarization effects

are considered to be a result of the scattering of the odd particle to the 1^+ excitations of the core.

The purpose of this work is to investigate the structure of these 1^+ states in even-even nuclei, their decay properties and the energy region in which they may be observed.

1. Hamiltonian

As was shown in more detail in ref.[3] the Hamiltonian describing the single-particle motion, pairing and magnetic dipole interaction in the quasi-particle representation is

$$H = H_{sqp} + H_{coll} + H_{int} , \tag{1}$$

where

$$H_{sqp} = \sum_{s,\tau} \epsilon_s(\tau) B_{ss}(\tau) \tag{1a}$$

$$H_{coll} = \frac{1}{4} \sum_{\tau,\tau'} \kappa_{\tau\tau'} \sum_{s,s'} \sigma^{(\mu)}_{ss'} L_{ss'} [C^+_{ss'}(\tau) + C_{ss'}(\tau)] \times$$

$$\times \sum_{m m'} \sigma^{(\mu)}_{m m'} L_{m m'} [C^+_{m m'}(\tau') + C_{m m'}(\tau')] . \tag{1b}$$

Here $\epsilon_s(\tau)$ are single-particle energies, $\tau \equiv \{n,p\}$. The operators $B_{ss'}$ and $C_{ss'}$ are expressed in the terms of the quasi-particle operators

$$B_{ss'} = \sum_{\rho=\pm} a^+_{s\rho}(\tau) a_{s'\rho}(\tau)$$

$$C_{ss'} = \frac{1}{\sqrt{2}} \sum_{\rho=\pm} a_{s'\rho}(\tau)\, a_{s,-\rho}(\tau) . \tag{2}$$

The state $|s,-\rho>$ is the time-reversed of $|s,\rho>$. The constants of the magnetic dipole interaction are chosen in the form

$$\kappa_{\tau\tau} = \kappa$$
$$\kappa_{np} \equiv q\kappa . \tag{3}$$

The magnitudes $L_{ss'} = u_s v_{s'} - u_{s'} v_s$ are expressed in terms of the parameters of the Bogolubov canonical transformation. Finally, in (1b) the quantities $\sigma^{(\mu)}_{ss'}$ are the single-particle matrix elements of the operator $\sigma_\mu + (-1)^\mu \sigma_{-\mu}$ $(\mu = 0, \pm 1)$, in which σ_μ are the Pauli spin matrices. In this way, in (1) the operator H_{sqp} describes the quasi-particle excitations, and H_{coll} describes the collective 1^+ - states with the projection of angular momentum $K=0$ or 1 ($\mu = 0$ or 1, respectively). H_{int} describes the interaction between the quasiparticles with these states and so far has not been related to our problem.

We remark, that because of the internal symmetry of the states with $K=0$; it is possible to build up a rotational band with spins $I = 1^+, 3^+, 5^+, ..$ etc. differing from the β-vibrational band.

As the two quasi-particle states, from which one builds 1^+ excitations, are usually higher than 2 MeV and are situated very close to one another, it seems reasonable to apply the T.D.Approximation.

The excited collective states are defined as one phonon states

$$|\Psi_1> \equiv Q_1^+ |\Psi_0> = \frac{1}{\sqrt{2}} \sum_{ss'\tau} \Psi_{ss'}(\tau)\, C^+_{ss'}(\tau) |\Psi_0>, \tag{4}$$

where $|\Psi_0>$ is the phonon vacuum, i.e. the ground state of the even-even nucleus, and $\Psi_{ss'}(\tau)$ are the amplitudes of the two-quasi-particle states, satisfying normalization condition

$$\sum_{ss',\tau} \Psi^{2}_{ss'}(\tau) = 1 \qquad (5)$$

and the symmetry condition $\Psi_{ss'} = -\Psi_{s's}$.

The dispersion equation for the excitation energy ω is obtained by the variational method in the form

$$(1+\kappa F_{\tau})(1+\kappa F_{\tau'}) - \kappa^{2} q^{2} F_{\tau} F_{\tau'} = 0, \qquad (6)$$

where

$$F_{\tau} = \sum_{ss'} \frac{\sigma^{2}_{ss'}(\tau)\, L^{2}_{ss'}(\tau)}{\epsilon_{ss'}(\tau) - \omega_{1}} \,. \qquad (7)$$

It is clear that F_{τ} has poles at $\omega_{1} = \epsilon_{ss'}(\tau)$. Because the constant κ is assumed to be positive, the solutions are expected between the two-quasi-particle energies, the lowest being above the energy gap.

For the two-quasi-particle amplitudes using the normalization condition, we obtain the following expression

$$\Psi^{1}_{ss'}(\tau) = \frac{\sigma_{ss'}(\tau) L_{ss'}(\tau)}{\epsilon_{ss'}(\tau) - \omega_{1}} \cdot \frac{1}{\kappa F_{\tau}} \cdot \frac{1}{\sqrt{\dfrac{Y_{\tau}}{\kappa^{2} F_{\tau}^{2}} + \dfrac{Y_{\tau'}}{(1+\kappa F_{\tau'})^{2}} q^{2}}} \qquad (8)$$

$$\Psi^{1}_{ss'}(\tau') = \frac{-\sigma_{ss'}(\tau) L_{ss'}(\tau)}{\epsilon_{ss'}(\tau) - \omega_{1}} \cdot \frac{1}{(1+\kappa F_{\tau'})} \cdot \frac{1}{\sqrt{\dfrac{Y_{\tau}}{\kappa^{2} F_{\tau}^{2}} + \dfrac{Y_{\tau'}}{(1+\kappa F_{\tau'})} \cdot q^{2}}},$$

$$Y_{\tau_i} = \sum_{ss'} \frac{\sigma^2_{ss'}(\tau) L^2_{ss'}(\tau)}{[\epsilon_{ss'}(\tau) - \omega_i]^2} . \qquad (9)$$

2. The Probabilities of M1 Transitions to the Collective 1^+ States

The characteristic quantity for the spin-vibrational 1^+ levels will be the probability for M1 transitions. Having in mind the expression (8) for the amplitudes of the states, we can calculate the matrix elements of M1 transition from the ground 0^+ state to excited 1^+ K 'states.

As is well known, the operator for M1 transition has the form

$$\mathfrak{M}_\mu(M1) = \sqrt{\frac{3}{4\pi}} \cdot \frac{D_\mu}{2} \cdot \frac{e\hbar}{2Mc}, \qquad (10)$$

where the components of the vector D_μ are defined as

$$D_\mu = g_s(\sigma_\mu + (-)^\mu \sigma_{-\mu}) + 2 g_\ell (\ell_\mu + (-)^\mu \ell_{-\mu}). \qquad (11)$$

Here, g_s and g_ℓ are respectively the spin and orbital g-factors of the nucleons. This part of the operator (10), whose matrix elements between the ground and excited states differ from zero, in the phonon representation has the form

$$\mathfrak{M}_{\mu}(M1) = \sqrt{\frac{3}{16\pi}} \cdot \frac{e\hbar}{2Mc} \sum_{ss'\tau} d^{(\mu)}_{ss'}(\tau)\, L_{ss'}(\tau) \times$$

$$\times \Psi^{i}_{ss'}(\tau) \cdot [Q^{+}_{i} + Q_{i}], \qquad (12)$$

where $d^{(\mu)}_{ss'}$ are matrix elements of the operator (11). Therefore, for the matrix element of the transition $0^{+} \to 1^{+}_{i}$ one obtains the expression

$$M_{i} = \langle \Psi_{0} | Q_{i}\, \mathfrak{M}_{\mu}(M1) | \Psi_{0} \rangle =$$

$$= \sqrt{\frac{3}{16\pi}} \cdot \frac{e\hbar}{2Mc} \sum_{ss'\tau} d^{(\mu)}_{ss'}(\tau)\, L_{ss'}(\tau)\, \Psi^{i}_{ss'}(\tau). \qquad (13)$$

The reduced probability for the M1 transition is defined as follows

$$B(M1, I_{i} K_{i} \to I_{f} K_{f}) = \langle I_{f} 1 K_{f} \mu | I_{i} K_{i} \rangle^{2} \cdot \frac{\langle I_{f} | \mathfrak{M}(M1) | | I_{i} \rangle^{2}}{2I_{i}+1} \left(\frac{2Mc}{e\hbar}\right)^{2}. \qquad (14)$$

For (14), taking into account (12) and (13) we obtain

$$B(M1, 0^{+}0 \to 1^{+}K) = \frac{3}{16\pi} \left| \sum_{ss'\tau} d^{(\mu)}_{ss'}(\tau) L_{ss'}(\tau)\, \Psi_{ss'}(\tau) \right|^{2}. \qquad (15)$$

3. The Sum Rule

It is well known that the magnitude of the matrix elements corresponding to the radiative transition is restricted by certain relations, called sum rule.

In the case of M1-transition the sum rule is written as follows[4]:

$$\langle \Psi_0 | [D_\mu [H D_\mu]] | \Psi_0 \rangle = \frac{8\pi}{3} \sum_i \omega_i B_i(M1), \qquad (16)$$

where $B_i(M1)$ are defined by expression (15). Let us calculate the double commutator entering (16). As D_μ commutes with H_{coll}, we have to find only the commutator $[H_{sqp}, D_\mu]$. After some manipulations we find

$$\langle \Psi_0 | [D_\mu [H, D_\mu]] | \Psi_0 \rangle = \frac{1}{2} \sum_{ss'\tau} [d^{(\mu)}_{ss'}(\tau)]^2 L^2_{ss'}(\tau) \epsilon_{ss'}(\tau). \qquad (17)$$

One must point out, the equality (16) reduces to an identity when $\kappa = 0$. Therefore the difference between the left- and right-hand sides of (16) is a measure of the contribution from the interaction which we are treating in the sum rule. By this difference one can estimate to some extent the degree of collectiveness or correlations of the two-particle interaction in the states under consideration[4]. The value of $B(M1)$ for each particular excitation will, as a rule, tell us something about the degree of collectiveness of this state. By means of the sum rule one can draw only qualitative conclusions about the degree of collectiveness of all the generated states, as was mentioned above.

It is interesting to consider the behaviour of the r.h.s. of (16) as a function of the number of the states involved in the sum, i.e. to consider the function

$$\chi_n = \frac{8\pi}{3} \sum_{i=1}^{n} \omega_i B_i(M1, 0 \to 1) \,. \qquad (18)$$

The behaviour of this function allows us to estimate the contribution of the various states in the sum rule and to determine the region of "saturation" where χ_n will be equal to the l.h.s. in (16).

4. Results of the Calculation and Discussion

We have done calculations for several rare earth nuclei: ^{156}Gd, ^{160}Dy, ^{168}Er, ^{170}Yb, ^{176}Hf and ^{182}W. The scheme of single-particle levels with 40 neutron and 40 proton states was used in the calculations[5]. The parameters of the pairing interactions were taken from the same work.

The number of the calculated two-quasi-particle 1^+ states turns out to be equal to 80 for the states with $K=0$ and 212 for the states with $K=1$.

The calculation have shown that all 1^+ states with $K=0$ up to energies of the order of 6-7 MeV are characterized by small values of the probabilities for M1-transitions from the ground state. As a rule, for these states $B(M1, 0 \to 1) / B(M1)_{s.p.} \lesssim 0{,}2$ [x].

---

x/ $B(M1)_{s.p.} = 1.79 \left[\left(\frac{e\hbar}{2Mc} \right)^2 \right]$

Among the low-lying states with $K=1$ there can exist states having the value of $B(M1, 0 \to 1)$ of the order of the single-particle value. As a rule, they are weakly collectivized but the large value for the matrix element of M1-transition is due to the strong contribution from the states of one of the subshells (at $\delta = 0$). In table 1 we show as an example the spectroscopically observed low-lying 1^+ states in ^{170}Yb /6/ and their interpretation. All the indicated states in the energy internal 2-3,5 MeV are weakly collectivized. The structure of their wave functions is in good agreement with the experimentally observed strong retardation for β-dacays of the type $0^+ \to 1^+$ x/. It is also characteristic that in the region of spectroscopic 1^+ states (at energies up to 4 MeV) there is usually a small number of states with $K^\pi = 0^+$. Therefore one can expect that the mixing of the states with $K=0$ and 1 due to the Coriolis interaction would be weak, as levels with suitable configuration are very seldom close-lying. Consequently, one can make use of Alaga rules for the intensities of the transitions in order to find the quantum number K of the 1^+ states.

It is interesting to establish the energy region in which strongly collectivized 1^+ states with large values of $B(M1)$ might appear.

In sperical nuclei the occurence of collective 1^+ states is connected with transitions between levels belonging to the spin-orbital partners/7/. In the nucleus ^{12}C such a state was observed at the energy 15.1 MeV by means of inelastic backward electron scattering/8/. Strong M1-transitions are also found in several light

x/ The parent nucleus ^{170}Lu has the ground state configuration $0^+\{n[633]\uparrow - p[404]\downarrow\}$. For all β-transition $\log ft$ has the characteristic magnitude ≈ 8, which is approximately 3 units larger than the usual value for allowed transitions.

nuclei in experiments on large-angle inelastic electron scattering (cf., for example, the review article[9]). In heavy spherical nuclei it is expected that collective spin- vibrational 1^+_1 states occur at lower energies (e.g. in ^{208}Pb a 1^+ state with a large value of B(M1) is expected at an energy of the order of 7.5 MeV[10]).

In deformed nuclei the picture is more complicated because of the mixing of states with different j . The selection rules for ℓ and j may be strongly violated. Strong M1-transitions can occur in the case when $\Delta K = 0$:

(i) between states originated from the spherical spin-orbital partners (e.g. $h_{11/2} \rightarrow h_{9/2}$). In the single-particle Nilsson scheme, the spacing between such levels is of the order of 8-10 MeV.

(ii) because of the j -mixing, in some cases strong M1-transitions will also occur between states originated from subshells with different ℓ (e.g. of the type $f_{7/2} \rightarrow h_{11/2}$, $f_{5/2} \rightarrow h_{9/2}$, $g_{9/2} \rightarrow i_{13/2}$ etc). Such transitions occur at lower energies and can form a collective 1^+0 -state with a magnitude of $B(M1, 0 \rightarrow 1) \approx \approx 0.4 B(M1)_{s.p.}$ (in the energy region of 7-8 MeV).

The characteristics of several collective 1^+0 states in ^{170}Yb are shown in table 2. The states 1^+0 having the largest values of $B(M1, 0 \rightarrow 1)$ occur for all rare earth nuclei in the region of 10-11 MeV.

In the case of $\Delta K = 1$, strong M1 transitions may be:

(i) between states originated from one spherical subshell (e.g between states of the subshell $h_{11/2}$ or $i_{13/2}$). Those states with the lowest transition energy may form the spectroscopically observed 1^+1 excitation with large values of B(M1) (cf. e.g. the state at energy 2.35 MeV in ^{170}Yb in table 1). But such excitations, as a rule, are weakly collectivized.

(ii) between states from the spin-orbital partners. The energies of such transitions are higher than those of the corresponding transitions in the case $\Delta K=0$ and usually are of the order of 12-13 MeV.

(iii) between states from subshells with different ℓ . These transitions also can form collective 1^+1 excitation at energies of the order of 7-9 MeV.

In table 3 we show the contributions of the transitions of various types in several 1^+1 states in ^{170}Yb . The calculation indicates that the 1^+1 states with the largest values of $B(M1, 0 \to 1)$ in rare earth nuclei occur in the region of 12-13 MeV.

Finally, we note that in low-lying 1^+ states (np) mixing, as a rule, is weak. Strong mixing of the states occurs in the region of the collective states.

In order to clarify the position of the magnitude dipole resonance, we carried out calculations of the magnetic dipole sum rule for some rare earth nuclei (cf. figs. 1-3). The solid curves show the "saturation" function χ_n i.e. the r.h.s. of the sume rule (16). It is clear that the major contribution to the sum rule comes from the states in the regions 10-11 MeV ($K=0$) and 12-13 MeV ($K=1$). The contribution of the 1^+ states in the region of resonance capture of neutrons is usually not greater than 10-15% of the total magnitude of the sum rule. Therefore we doubt the possibility of observing the magnetic dipole resonance in the (n,γ) reactions, as is suggested in ref.[11], though some collective 1^+ states can appear in the energy region of the resonance capture of neutrons (cf. e.g. (12, 13).

Inelastic electron scattering at large angles and resonance photoabsolution or (γ,n) reactions obviously, may be a convenient method of looking for M1-resonance.

Some results of this work have been presented at the conference in Montreal[14] and have been reported to a seminar in the International School of Theoretical Nuclear Physics in Predeal (Romania, September, 1969).

In conclusion, the authors express their gratitude to the members of Nuclear Theory Department for useful discussions. One of the authors (S.G.) is grateful to Prof. G.Alaga for his interest in this work and for useful discussion.

References

1. Z.Bochnacki, S.Ogaza. Nucl.Phys., 69, 186 (1965); 83, 619 (1966); A102, 529 (1967).
2. A.B.Migdal. Nucl.Phys., 75, 441 (1966).
3. L.P.Rapoport, A.S.Chernyshev. Yader. Fiz., 7, 309 (1968).
3. A.A.Kuliev, N.I.Pyatov. Yader. Fiz., 9, 313, 955 (1969). A.A.Kuliev, N.I.Pyatov. Phys.Lett., 28B, 443 (1969).
4. R.A.Broglia, A.Molinary, B.Sørensen. Nucl.Phys., A109, 353(1968).
5. K.M.Zheleznova, A.A.Korneichuk et al. Preprint D-2157, Dubna, 1965.
6. N.A.Bonch-Osmolovskya , G.Balund, Z.Prejbish et al. Preprint JINR P6-4773, Dubna, 1969.
7. B.R.Mottelson. Proc. Intern. Conf.Nucl.Str., ed, by D.A.Bromley, and E.W.Vogt, Kinston, Canada, 1960, p.525.
8. J.Goldemberg, W.C.Barber, F.H.Lewis, J.D.Walecka. Phys.Rev., 134, B1022 (1964).
9. H.Überall. Springer Tracts in Mod. Phys., v.49, Springer-Verlag, Berlin, 1969, p.1.
10. R.A.Broglia, A.Molinari, B.Sørensen. Nucl.Phys., A109, 353 (1968).

11. C.S.Shapiro, G.T.Emery. Phys.Rev.Lett., 23, 224 (1969).
12. L.M.Bollinger. Nuclear Structure, Dubna Symposium , 1968. IAEA, Vienna, 1968, p.317.
C.Samour, ibid. p.361.
13. F. Bechvarj, Ya.Vrzal, Ya.Liptak, Ya.Urbanetz. Preprint P3-3696, Dubna, 1968.
14. S.Gabrakov, A.A.Kuliev and N.I.Pyatov. Contributions, Int. Conf. Prop.Nucl.St., Montreal, Canada, 1969, p.39.

Received by Publishing Department
on November 10, 1969.

Table I.

Characteristics of low-lying I^+ states in ^{170}Yb.

Energy (MeV)	K^π	$\frac{B(MI,0\to I)}{B(MI)_{sp}}$	STRUCTURE		Experimental[6] Energy (MeV)	Experimental[6] K^π
2,35	1^+	1,2	98%	pp $\{[514]\uparrow - [523]\uparrow\}$	2,039	1^+
2,43	1^+	$4{,}10^{-3}$	98%	nn $\{[633]\uparrow - [642]\uparrow\}$		
2,50	0^+	0,03	99%	nn $\{[523]\downarrow - [512]\uparrow\}$	2,533	0^+
2,61	1^+	0,13	97%	pp $\{[411]\uparrow - [411]\downarrow\}$	2,641	?
2,72	1^+	≈ 0	100%	pp $\{[404]\downarrow - [402]\uparrow\}$	2,783	1^+
2,74	1^+	10^{-5}	100%	nn $\{[514]\downarrow - [512]\uparrow\}$		
2,81	1^+	0,03	78%	nn $\{[521]\uparrow - [521]\downarrow\}$	2,883	?
			13%	nn $\{[512]\uparrow - [521]\downarrow\}$		
			5 %	nn $\{[624]\uparrow - [633]\uparrow\}$		
2,93	1^+	0,01	77%	nn $\{[512]\uparrow - [521]\uparrow\}$	2,965	1^+
			17%	nn $\{[624]\uparrow - [633]\uparrow\}$		
2,99	1^+	0,3	86%	nn $\{[624]\uparrow - [633]\uparrow\}$	3,092	1^+
			13%	pp $\{[404]\downarrow - [413]\downarrow\}$		
3,03	1^+	$2{,}10^{-3}$	74%	nn $\{[624]\uparrow - [633]\uparrow\}$	(3,148)	(0^+)
			8%	nn $\{[512]\uparrow - [521]\uparrow\}$		
			8%	nn $\{[514]\downarrow - [523]\downarrow\}$		
3,20	1^+	0,01	88%	nn $\{[514]\downarrow - [523]\downarrow\}$	3,184	?
			3 %	nn $\{[624]\uparrow - [633]\uparrow\}$		
3,33	1^+	10^{-7}	100%	nn $\{[523]\downarrow - [521]\uparrow\}$		
3,35	1^+	0,6	99%	pp $\{[402]\uparrow - [411]\uparrow\}$	3,302	?
3,50	1^+	3.10^{-3}	98%	nn $\{[521]\uparrow - [510]\uparrow\}$		
3,52	0^+	0,02	99%	nn $\{[521]\downarrow - [510]\uparrow\}$		

Table 2.

Structure of several I^{π} K = $I^{+}0$ states in ^{170}Yb in the region of high energies. The states with the largest values of B(MI) are given.

ω (MeV)	$\frac{B(MI,0\to I)}{B(MI)_{sp}}$	STRUCTURE	Amplitudes $\psi_{ss'}$	Shell transitions (δ =0)
7.77	0.36	pp 530↑- 550↑	-0.605	$h_{9/2} \to h_{11/2}$
9.10	0.35	nn 541↓- 510↑	-0.291	$f_{7/2} \to f_{5/2}$
		nn 651↑- 631↑	0.272	$i_{11/2} \to i_{13/2}$
		pp532↑ - 523↓	0.421	$h_{11/2} \to f_{7/2}$
		pp 523↑- 514↓	-0.296	$h_{11/2} \to f_{7/2}$
10.35	1.24	nn 550↑- 521↓	-0.496	$h_{11/2} \to p_{3/2}$
		nn 651↑- 642↓	-0.163	$i_{13/2} \to g_{9/2}$
		nn 651↑- 631↑	-0.173	$i_{13/2} \to i_{11/2}$
		nn 532↑- 523↓	-0.220	$h_{11/2} \to f_{7/2}$
		nn 523↑- 503↑	-0.134	$h_{11/2} \to h_{9/2}$
		pp 541↑- 521↑	-0.173	$h_{11/2} \to h_{9/2}$
		pp 523↑- 514↓	0.130	$h_{11/2} \to f_{7/2}$
10.89	0.47	nn 541↑- 521↑	0.425	$h_{11/2} \to h_{9/2}$
		pp 541↑- 521↑	0.531	$h_{11/2} \to h_{9/2}$

Table 3

Structure of several $I^{\pi} K = I^{+}1$ states in ^{170}Yb in the region of high energies. The states with the largest values of B(M1) are given.

ω (MeV)	$B(M1)/B(M1)_{s.p.}$	Structure	Amplitudes $\psi_{ss'}$	Shell transitions ($\delta = 0$)
		pp 411↑ – 400↑	-0.183	$g_{7/2} \rightarrow d_{3/2}$
		pp 541↑ – 541↓	0,301	$h_{11/2} \rightarrow f_{7/2}$
		pp 413↓ – 402↓	-0,165	$d_{5/2} \rightarrow d_{3/2}$
5,26	0,64	pp 532↑ – 532↓	-0,493	$h_{11/2} \rightarrow f_{7/2}$
		pp 404↑ – 404↓	0,178	$g_{9/2} \rightarrow g_{7/2}$
		pp 420↑ – 411↓	0,163	$g_{7/2} \rightarrow s_{1/2}$
		nn 402↓ – 411↓	-0,537	$d_{3/2} \rightarrow s_{1/2}$
12,72	1,29	nn 532↑ – 501↑	0,110	$h_{11/2} \rightarrow f_{5/2}$
		pp 514↑ – 503↑	-0,359	$h_{11/2} \rightarrow h_{9/2}$
		nn 550↑ – 530↑	-0,310	$h_{11/2} \rightarrow h_{9/2}$
		pp 422↓ – 440↑	0,127	$d_{5/2} \rightarrow g_{9/2}$
12,93	1,02	pp 512↑ – 541↑	-0,555	$h_{9/2} \rightarrow h_{11/2}$
		pp 514↑ – 503↑	0,167	$h_{11/2} \rightarrow h_{9/2}$
		nn 642↑ – 642↓	-0,136	$i_{13/2} \rightarrow g_{9/2}$
		nn 523↑ – 503↓	0,160	$h_{11/2} \rightarrow f_{5/2}$
		nn 514↑ – 514↓	-0,166	$h_{11/2} \rightarrow f_{7/2}$
		nn 505↑ – 505↓	-0,153	$h_{11/2} \rightarrow h_{9/2}$
		nn 550↑ – 530↑	-0,121	$h_{11/2} \rightarrow h_{9/2}$
		pp 422↓ – 440↑	-0,117	$d_{5/2} \rightarrow g_{9/2}$
13,07	6,02	pp 512↑ – 541↑	0,320	$h_{9/2} \rightarrow h_{11/2}$
		pp 523↑ – 523↓	0,137	$h_{11/2} \rightarrow f_{7/2}$
		pp 514↑ – 503↑	0,216	$h_{11/2} \rightarrow h_{9/2}$
		pp 505↑ – 505↓	-0,103	$h_{11/2} \rightarrow h_{9/2}$
		pp 440↑ – 420↑	0,236	$g_{9/2} \rightarrow g_{7/2}$

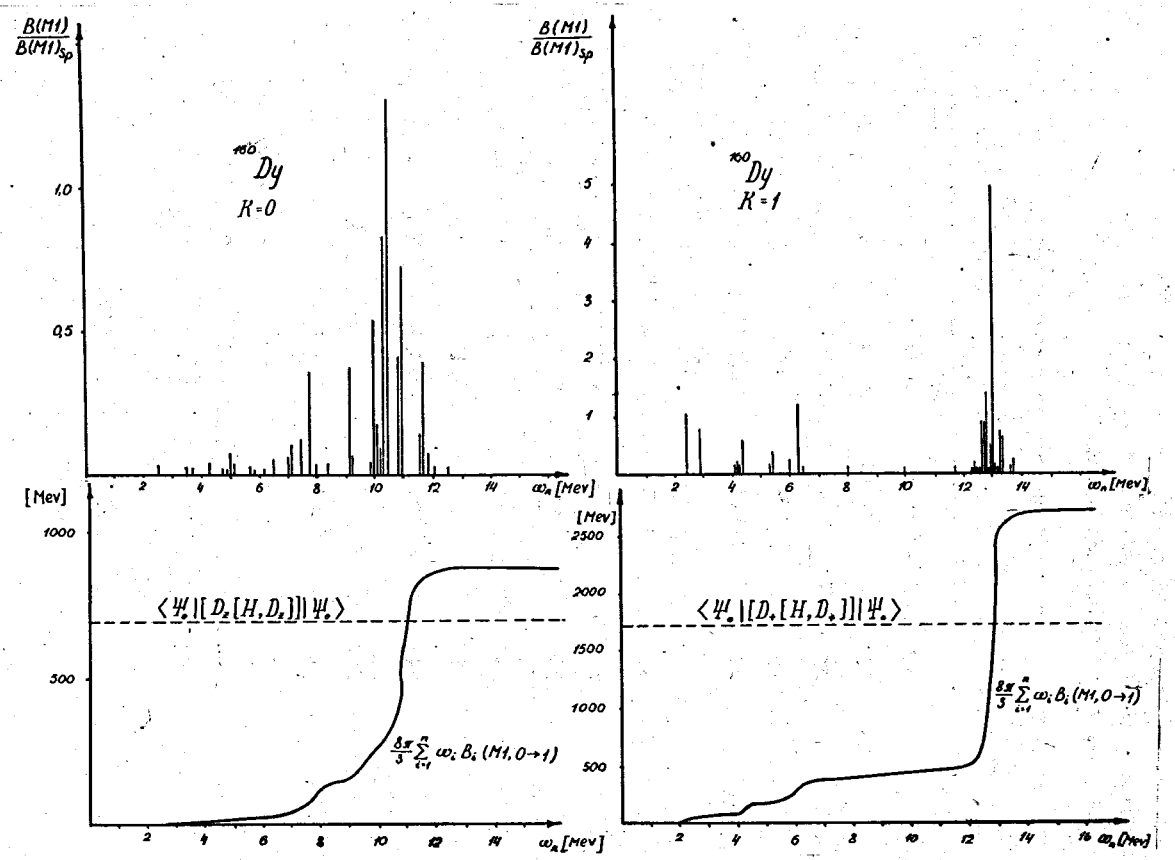


Fig.1. Energy diagrams, values of B(M1) and sum rules for 1^+ states with K=0 (on the left) and K=1 (on the right) in ^{160}Dy . . Single-quasi-particle estimate of the sum rule is shown by dashed line. The solid curve corresponds to the function χ_n .

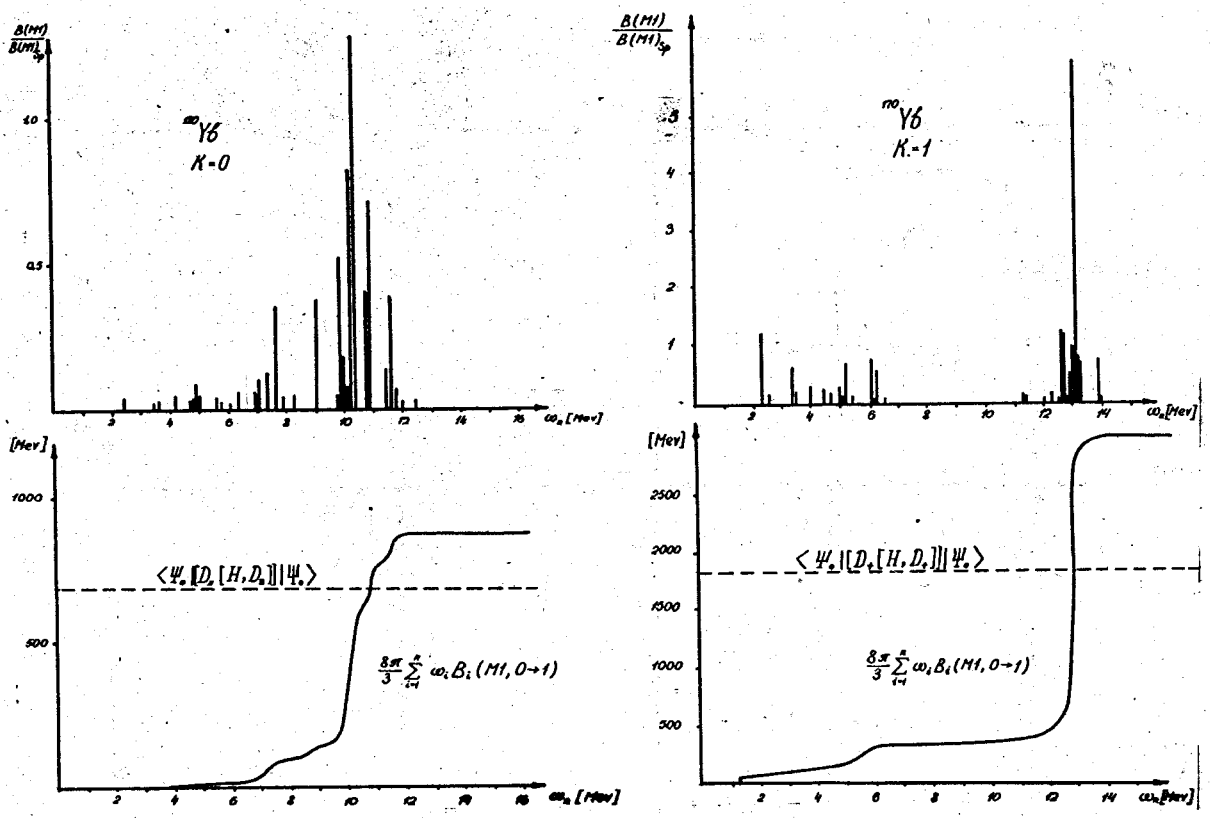


Fig.2. The same as in Fig.1, for the nucleus ^{170}Yb .

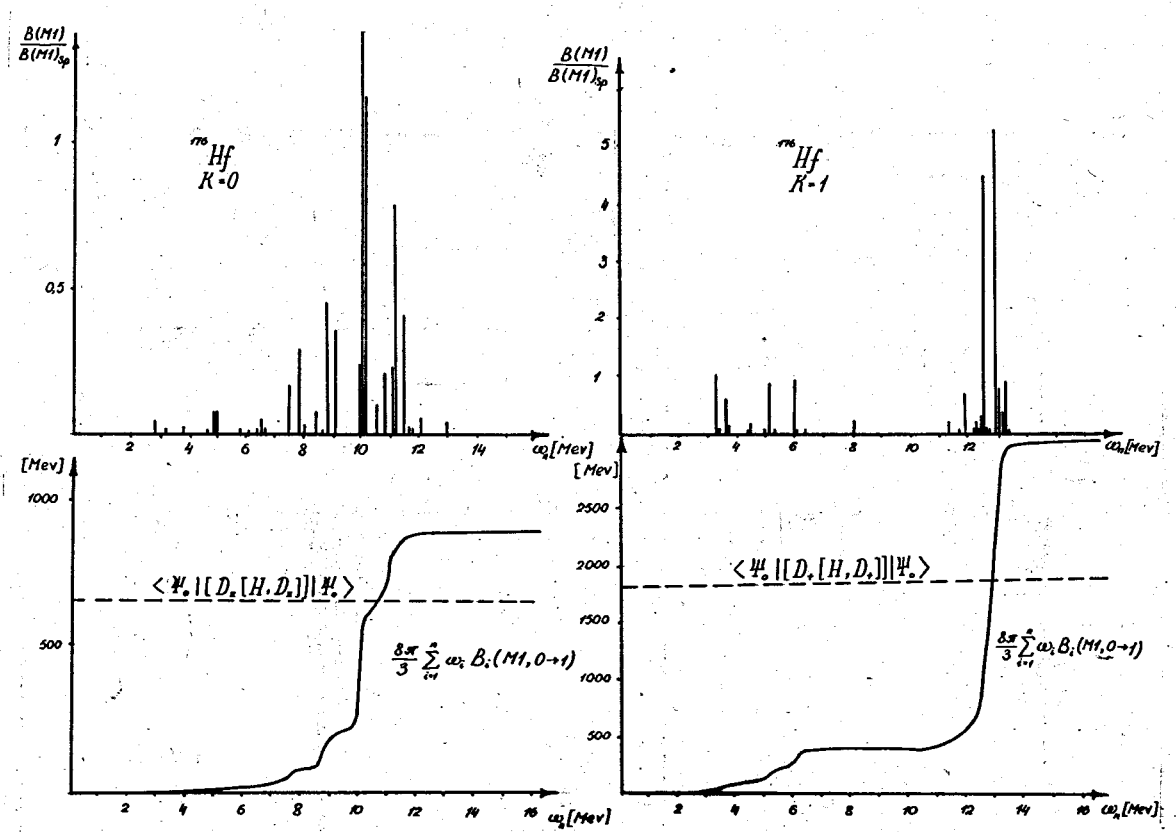


Fig.3. The same as in Fig.1, for the nucleus ^{176}Hf .