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A.M.Baldin

**HEAVY ION INTERACTIONS
AT HIGH ENERGIES**

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Invited Talk given at the VI International
Conference on High Energy Physics and
Nuclear Structure (June 9, 1975,
Santa Fe, USA)

I shall make a review mainly on the results of research carried out at JINR, Dubna.

Special attention will be given to single-particle inclusive processes, which occur when relativistic nuclei interact, since more complicated processes are not studied well enough at present. In addition, I shall make a short review of the program of research with relativistic nuclei at the Laboratory of High Energies, JINR.

PHYSICAL MOTIVATION

The methods to describe composite systems, for which relativistic effects are important, are of particular concern nowadays. First of all, quark-parton models and the methods used to describe hadron structure should be mentioned.

Already by the end of the fifties, physicists refused to accept the assertions, widely used in text-books, that the elementary particle in principle had no dimensions.

At present we know well the size of the space region occupied by protons. We also know that the density of matter in this region is only three times larger than that of nuclear matter. The distances between nucleons in a nucleus are comparable to their size. We can only be astonished at the success of nuclear models in which the nucleus is considered as a set of point-like objects.

In modern accelerators we deal with wave lengths which are more than 1000 times less than the size of the proton.

However, as long ago as the beginning of the sixties M.A. Markov stressed that the removal of difficulties of the quantum field theory meant that we would need another concept of elementary size.

He noted^{/1/} that this factor was connected with abundant production of other particles and suggested the idea that the inelastic form factor of particles had properties which were characteristic of scattering on point particles. This idea was verified in deep inelastic scattering processes of electrons on protons. The deep inelastic scattering processes posed an age-long question: What next? i.e., what structure units should be taken as a base of "internal proton structure". The concept of new constituents with infinitesimal sizes, partons, is developing rapidly. True, instead of formulae, pictures resembling abstract art are usually drawn. The quark model holds a better position. This model was developed from composite models and has on its credit side not only verified predictions, but also regularities (formulae!) which involve a very wide class of experimental material.

Composite models, as an initial hypothesis, use the concept of the existence in nature of binding energies comparable to the masses of constituent particles.

The latter implies an essential relativistic approach to the problem of bound states, which in itself represents a fundamental problem. In any case the problem of describing "interhadron matter", and the related problem of the relativistic description of

extended composite objects, face physicists in all its magnitude. Due to a rapid accumulation of experimental data, the situation in this important field is becoming critical and resembles the state of the theory of the structure of matter during Rutherford's famous experiments.

Just as the α -particle large angle scattering on gold found by Rutherford showed the existence of elementary constituents inside the atom, a large momentum transfer in the pp large angle scattering shows the existence of such objects inside the nucleon. Physics of relativistic ion interactions, relativistic nuclear physics, is, as a matter of fact, a new approach to the same group of problems. The approach to the problems in relativistic nuclei physics is a natural generalization of that in elementary particle physics: 1) study of elastic scattering in the region of extremely small (up to the Coulomb interference), and extremely large momentum transfers; 2) investigation of various features of multiple particle production in the interaction of relativistic nuclei; 3) research on regularities, of the scale invariance type, in application to composite systems.

Moreover, the collision of relativistic nuclei is richer in forms and more informative. That gigantic energies are concentrated not at a point but in significant space regions is an important feature of the interaction of relativistic nuclei. These unique conditions must result in important consequences. The concept of a continuous medium must manifest itself.

We define relativistic nuclear physics as the field of many-baryon phenomena given by the condition

$$\frac{P^2}{m^2} \gg 1, \quad (1)$$

where P^2 are the particle momenta squared and m^2 are their masses squared.

A theoretical consideration is significantly simplified due to the fact that the scale invariance is applicable in this field. Scale invariance is one of the most important laws which characterize "interhadron matter". A comprehensive check of this law in the interactions of composite systems (partly made by us) is of great interest. I shall deal with this question below.

The condition (1) makes it possible to consider the asymptotics of matrix elements. In particular, it enables one to introduce one of the criteria which determines the cumulative effect.

We understand the cumulative effect as the process of interaction of a relativistic nucleus with a target. As a result energy, which significantly exceeds that per nucleon of the incident nucleus, is transferred to the produced particles. We focus our attention on this phenomenon. The cumulative effect is a very striking but only one of the many multiple production processes which occur in the interaction of relativistic nuclei. Multiple production processes accounting for the majority of all high energy reactions are now the most intensively studied processes in high energy physics. It is obvious that a tendency to increase the significance of research into multiple production processes will remain for many years because these processes are complicated and multi-form. A study of the interaction of relativistic nuclei permits a nontrivial approach to many-body processes. Very important features which characterize the investigation of the interaction of relativistic nuclei in comparison with the collision of particles, are the following:

1) The internal structure of interacting objects is known at least in the nonrelativistic limit.

2) One can vary the quantum numbers of colliding objects within wide limits.

3) It is possible to study multiple processes when there are many particles not only in the final but also in the initial state of the process (cumulative effects).

4) A more justified use of the statistical and hydrodynamic approaches is possible (there is a much larger number of configurations over which the averaging is made).

In addition to inelastic interactions and multiple production processes, a study of the behaviour of total nucleus-nucleus cross sections (in particular, factorization), binary reactions, elastic scattering with large momentum transfers, is of great interest. These reactions are interesting from the viewpoint of testing a number of models used in elementary particle theory, since the nuclei can serve as a realistic "quark" model of the relativistic extended object. The structure of our objects may be varied over wide limits by choosing different beams and targets. It is possible to explain the cumulative effect as a many-quark interaction involving large distances compared to a nucleon size. Quark degrees of freedom for the nucleus turned out to be important for large momentum transfer, and the cumulative effects can clarify the large distance interaction between quarks. This point is essential for clarifying the problem of quark confinement.

MAIN DEFINITIONS AND VARIABLES

As we deal with the field of phenomena defined by the condi-

tion (1), it is necessary to stress that experimental data should be considered and presented in the completely relativistic invariant form. The use of a noninvariant approach leads, as will be seen below, to some difficulties and even to apparent contradictions.

In order to describe the inelastic processes (single-particle distributions)

$$I + II \rightarrow 1 + \dots$$

we use the relativistic invariant sum of the cross sections having the same initial state and one particle in the final state with the given characteristics

$$f = \sum_n E_1 \frac{d\sigma^n}{d^3p_1} \quad (2)$$

It is convenient to introduce as invariant variables somewhat different from those used in elementary particle physics

$$v = \frac{P_x \cdot P_x}{m_I}; \quad b_{I1} = \frac{P_x \cdot P_1}{m_I} - m_1; \quad m_{I1} = \sqrt{P_{I1}^2 + m_1^2} \quad (3)$$

Here m_1 is the particle mass, P_{I1} is the projection of the three-dimensional momentum of particle 1 on the plane perpendicular to the reaction axis (to the direction of the collision of nuclei I and II)

$$f = f(v, b_{I1}, m_{I1}^2) \quad (4)$$

The following considerations show that it is worthwhile to introduce these variables^{1/2}. We fix our main attention on the variable region in which nuclei I and II are in unequal positions.

$$(P_x \cdot P_1) \ll (P_x \cdot P_1) \sim (P_x \cdot P_x) \quad (5)$$

This region corresponds to the limiting fragmentation of Yang et al.

$$f = f(v, b_{I1}, m_{I1}^2) \Big|_{v \rightarrow \infty} = f(b_{I1}, m_{I1}^2) \quad (6)$$

The dependence on b_{I1} is also the dependence on the known scale variable $x = P_x^1 / P_I$, where P_x^1 is the projection of the three-dimensional momentum of particle I on the direction of the reaction axis; P_I is the three-dimensional momentum of particle I. In fact, using the condition (1), we have in the rest frame of nucleus II

$$b_{I1} = \frac{m_I x}{2} + \frac{m_{I1}^2}{2m_I x} - m_1 \quad (7)$$

Low binding energy of constituents is one of the distinctive features of the interaction of nuclei in comparison with that of particles. This leads to a large role of stripping and pickup.

These processes are described by an ordinary pole approximation. In this approximation the amplitude of the reaction $I + II \rightarrow 1 + \dots$, which proceeds via the single-particle intermediate state with mass m_2 , takes the form

$$T_{fi} = \sum_j \frac{T_{ij} T_{ji}}{(P_i - P_j)^2 - m_2^2} \quad (8)$$

Elementary transformations permit one to separate from the relativistic invariant cross section the denominator expressed in terms of b_{I1}

$$\frac{d\sigma}{db_{I1}} = \frac{F}{\left(x + \frac{2}{m_I} b_{I1}\right)^2} \quad (9)$$

where $\xi = \frac{(m_1 + m_2 - m_1)(m_1 + m_2 - m_1)}{m_1 m_1}$. (10)

The fraction with such a denominator has the character of a ξ -function on the variable b_{r1} . This is due to the fact that the parameter α is very small, either because the number of nucleons in nucleus I is equal to the sum of the numbers of nucleons in nuclei 1 and 2 (stripping reaction), or because the number of nucleons in nucleus 1 is equal, respectively, to the numbers of nucleons in nuclei I and 2. It is interesting to note that the binding energy per one nucleon cancels out the differences enclosed in parentheses. The analysis of nuclear fragmentation reactions in the relativistic region is essentially simplified by introducing one parameter b_{r1} instead of longitudinal and transverse momenta.

There is a limit on b_{r1} resulting from the conservation laws. In a $(\frac{m_1 m_2}{v})^2 \ll 1$ approximation, which corresponds to the condition (1), the conservation laws expressed in terms of invariants take the form

$$b_{r1} = \frac{m_1}{2} \left[1 - \frac{\Delta}{2\sqrt{M_1^2}} + \frac{m_{11}^2}{m_1^2 (1 - \frac{\Delta}{2\sqrt{M_1^2}})} \right] - m_1, \quad (11)$$

where $\Delta = M_1^2 - m_1^2 - m_2^2 - m_1^2$; $M_1^2 = (P_1 + P_2 - P_1)^2$.

The maximum value of b_{r1} is determined by the minimum value of Δ (or M_1^2).

Assuming that eq. (11) can be referred to the part of nucleus I (fragmenting nucleus): $P_I \rightarrow \lambda P_I$; $m_I \rightarrow \lambda m_I$, for $\Delta = \Delta^{\min}$ we obtain the minimum value of the number $\lambda < 1$ corresponding to the measured value of b_{r1} . The λ parameter defines what we call cumulativity. If $\lambda^{\min} > \frac{2}{A_I}$ (A is the atomic number),

according to our assumption more than one nucleon of nucleus I take part in the interaction. The value $N = \lambda A$, the effective number of nucleons inside nucleus I participating in the collision, is called cumulativity order. The particle production characterized by $N \gg 1$ is called the cumulative effect. One can understand our assumption either as a consequence of scale invariance or as a consequence of a composite nature of nuclei (in the last case $N = \lambda A$, the number of nucleons).

The b_{r1} distribution of reaction products makes it possible to classify the interactions of relativistic nuclei. The group of reaction products in a small vicinity of b_{r1} near zero ($b_{r1} \leq \xi$, where ξ is the nucleus binding energy) should be referred to "fragments", the part which is due to the peculiarities of the nucleus as a weakly bound system. In the region of large values, the measured value of b_{r1} determines the minimum value of the cumulativity parameter N (within the framework of our assumption):

$$b_{r1} = \frac{Nm_p}{2} \left[1 - \frac{\Delta^{\min}}{2\sqrt{M_1^2}} + \frac{m_{11}^2}{m_1^2 (1 - \frac{\Delta^{\min}}{2\sqrt{M_1^2}})} \right] - m_1, \quad (12)$$

where m is the nucleon mass.

This is a formal definition of the cumulative effect. In order to prove the existence of the effect of interaction of nucleon groups, it is necessary to consider the concepts used in the model used to describe the mechanism of interaction of relativistic nuclei. In particular, our definition of cumulative effects neglects the Fermi motion.

RESULTS ON THE CUMULATIVE EFFECT

By the end of 1970, when these works were started, it was

known that the scale invariance appeared not only in electron-proton but also in hadron interactions.

One of its possible interpretations consists in the fact that for very high energies the mass, form factor and other constants of length dimension are inessential, and the "interhadron matter" represents a homogeneous medium (similar to a point explosion picture - automodelity).

We have practically assumed^{/3/} that for high energies the group of nucleons is also a homogeneous solid medium if the distance between them is less or if the order of characteristic length $\rho \sim \frac{1}{R} 0.7 \cdot 10^{-13}$ cm. For such a "little drop"^{/16/} ("core") scale invariance should appear when it interacts, e.g., with a nucleon. In this case the cross section is factorized into two terms which determines the probability of finding such a "drop" inside the nucleus and into the scale invariance function which is taken to be universal (independent of the number of nucleons in the core) and to be equal to the structure function of meson production in pp interactions. Using this model, we predicted the ratio of the meson production cross sections for the reactions

$$\left. \begin{array}{l} \alpha + A \rightarrow \pi^- + \dots \\ \rho + A \rightarrow \pi^- + \dots \end{array} \right\} (I + II) = 1 + \dots \quad (13)$$

For proton energies which exceed by a factor of two the energy per nucleon in the deuteron and for the parameter $\chi_d \approx \frac{R}{R_d} > 0.5$, it was expected that the ratio would be $\sim 5-10\%$. This means that for a deuteron momentum of 8 GeV/c (4 GeV/c per nucleon) one has a large probability to obtain 5-7 GeV pions. This prediction did not seem probable for many physicists. Nevertheless, this estimate was completely supported by the experiment^{/4/}.

We found the pions carrying away up to 98% of the deuteron kinetic energy. It is important that the experiment demonstrated the application of the scale invariance to composite (nuclear) systems. The group of V.S. Stavinsky has been studying the cumulative effect, and below I shall present some experimental data on this interesting phenomenon accumulated at the present time.

The most obvious question is: Is it possible or not to explain the effect by the Fermi motion?

The calculations with relativistic invariant models^{/8/} in which involved different models for the deuteron form factor (including the form factor found in the electron-deuteron scattering) could not explain the observed effect even in the order of magnitude. Since then two papers have appeared: in one of them^{/9/} our conclusion that it is impossible to explain the effect by the Fermi motion was supported (for relativistic nitrogen nuclei), but in the second paper^{/10/} (for relativistic deuterons) both the results of calculation^{/8/} and the experimental data^{/4/} were not supported.

It is difficult to compare the calculations of papers^{/10/} and ^{/8/} because in paper^{/10/} only the results of calculations are presented.

My personal view is that explanations of large b_{11} effects by the Fermi motion should be considered on the same footing as for example the attempt to describe the deep inelastic scattering with the help of one intermediate particle diagram. As to the experimental data, it is necessary to present them for the same energy of incoming deuterons and, perhaps, it is more important to have unified data presentation at least in the same frame. Figure 1 presents the data of refs. ^{/8/} and ^{/10/} as well as

of ref.^{/11/} as a function of $\mathcal{E} = \frac{b_{Li}}{b_{Li}^{max}} = \frac{T'}{T^{max}}$ where T is the pion kinetic energy in the antilab. coordinate system. As is seen from the figure a larger decrease in the cross section with increasing the invariant value \mathcal{E} (or \mathcal{L}) (see paper^{/10/}) is not observed

However, the model proposed by us^{/3/} has a prediction which cannot be explained by the Fermi motion and which is well supported by a large number of experiments carried out by the group of V.S. Stavinsky.

According to the model, the function f reads

$$f_{II}^I = \sum_{N=1}^A P_N f_N(\mathcal{E}_N), \quad (14)$$

where P_N , the probability of interaction of N nucleons in a nucleus, can be taken as a binomial distribution.

$$P_N = \frac{A!}{N!(A-N)!} q^N (1-q)^{A-N},$$

where for the probability of a single interaction q it is natural to consider two cases:

(1) q is determined as a probability of finding the nucleon in the space $4/3 (\pi r^3)$: $(\frac{r}{r_0} A^{4/3})^3 \sim 1/A$ (2) q is determined as a probability of finding the nucleon in the area $2\pi r^2$:

$q \sim (\frac{r}{r_0} A^{4/3})^2 \sim 1/A^{2/3}$. The last case corresponds to a strong increase of the dependence on the atomic number with increasing the cumulative number (approximately an additional factor $A^{1/3}$ per each order of cumulativity).

In formula (14) we stressed by indices different roles played by nuclei I (upper index) and II (lower index). As is shown in paper^{/9/}, the dependence of f_{II}^I on the atomic number of target nucleus A_{II} is weak ($A_{II}^{1/3}$). This is in agreement with our understanding^{/11/} and was also supported in ref.^{/10/}. A

study of the dependence of f_{II}^I on A_I in a wide range of A implies accelerating heavy nuclei, which is a complicated acceleration problem. In connection with this, we started to investigate the cumulative effect in the antilaboratory coordinate system. The description of the installation (fig. 2) is given in paper^{/5/}. Nuclei, including heavy ones, are bombarded by accelerated protons, neutrons and α -particles. The kinetic energy of particles produced in the backward direction, i.e., at an angle of 180° to the primary beam, was measured.

In the rest frame of nucleus I: $\epsilon = \epsilon_{II}^I$ is the energy of bombarding particles and $b_{II} = T$ is the kinetic energy of produced particles. So the relativistic invariant quantities defined above were directly measured, and it is possible now to investigate the cumulative effect practically for any nuclei.

The following isotopes (playing the role of nucleus I) ${}^6\text{Li}$, ${}^7\text{Li}$, C, Al, Cu, ${}^{112}\text{Sn}$, ${}^{124}\text{Sn}$, ${}^{144}\text{Sm}$, ${}^{154}\text{Sm}$, ${}^{182}\text{W}$, ${}^{186}\text{W}$, Pb, U were investigated. To compare the experimental data with previous ones, the deuteron cumulative effect was measured by this method and agreement was obtained (see fig. 1 discussed above).

The basic results^{/5-11/} illustrated in figs. 1-10 can be summarized as follows:

(1) The peculiarities of the form factor structure, number of neutrons and the surface shape of nuclei do not play a significant role in cumulative meson production. The ratio of cumulative π^- to π^+ is close to 1. These results support the viewpoint that the cumulative effect manifests the local properties of nuclear matter.

(2) The transition to limiting fragmentation (independence of the cross section on γ) takes place already in the region 4 GeV/nucleon (see fig. 4). It should be noted that data below 2 GeV have been obtained by extrapolation to $\theta = 180^\circ$ of data from the literature and must be checked.

(3) The bulk of data on the spectra of cumulative pions is expressed well in terms of a simple formula:

$$f_{\pi}^{\pm} = C \cdot A_I^n \cdot A_{II}^{4/3} \exp\left[-\frac{E_{\pi}}{E_0}\right] \quad (15)$$

where $n \geq 1$ at $N \geq 2$; b and $c = \text{const}(\gamma)$ $b_0 \approx 60$ MeV

The secondary beams from large fluxes of relativistic nuclei (including heavy nuclei) are represented by the formula (15). The figures (5,6,7) show this dependence in the antilaboratory coordinate system as a function of meson kinetic energy.

$$T_{\pi} = E_{\pi} - m_{\pi} = E_{I1}$$

The transition from the $A^{2/3}$ dependence to the A^n dependence is illustrated by figs. 7,8 and 10. Figure 9 illustrates the strengthening of the A_I dependence with increasing the cumulative number. It should be noted that powers A_I and A_{II} differ almost in order of magnitude.

(4) The cumulative effect up to the 4th order of magnitude (4 nucleons take part in the collision) turns out to be rather observable. This points not only to the possibility of its comprehensive study but also to some practical application. In particular, a strong dependence on A permits one to state that, at equal intensities of the circulating beam of protons and carbon nuclei in the specified accelerator, the intensities of the secondary beam for carbon nuclei will be higher despite the fact that

in the last case the energy per nucleon is less by a factor of two. For low intensities, particles with an energy significantly exceeding the nominal energy of the accelerator, can be produced.

(5) At equal energies per nucleon the fragmentation of nuclei on protons and neutrons is identical within the errors of our experiments.

G.A.Leksin presented to this Conference an analysis of all data available in the literature, and data of his group, on backward production of particles from nuclei. The selfconsistency of all these data is remarkable.

All of them can be described by formula of the type of (15) (See also^{/17/}).

A very interesting approach to the cumulative effect, based on parton model, was developed by N.N.Nikolaev and V.I.Zakharov^{/20/}. They do not agree with our model and suggested that the observation of the cumulative effect is a proof of final state interaction of partons. But it would be difficult to explain striking A -dependences observed by us with this suggestion.

The concept of expanding hadron cluster was introduced by B.N.Kalinkin and V.L.Shmonin^{/18/} in an attempt to explain important characteristics in hadron-nucleus interaction. The motion of the cluster in nuclear matter produces, in some cases, a new phenomenon: the generation of a shock wave, resulting in the complete decay of the heavy nucleus. Actually this phenomenon was observed earlier at Dubna^{/19/}. The predictions of the expanding hadron cluster model are:

1) the rate of the total decay ($n_n \gg 28$) should increase with the mass number A of the incident nucleus. At not very large

A_{II}^- the partial cross section is proportional to $\sim A_{II}^2$.

2) The multiplicity of relativistic particles should increase somewhat weaker than A_{II}^- .

3) The form of the distribution over the number of slow particles n_{II} should depend weakly on A_{II}^- and at $A_{II}^- = \text{const}$ to the first approximation it should not be changed with increasing energy E_{inc} .

4) With increasing A_{II}^- the relation between the number of g - and b -particles should change in favour of the first ones.

These predictions should be verified by more refined calculations.

RELATIVISTIC NUCLEAR PHYSICS AT DUBNA

At present the Dubna physicists from LHE (Laboratory of High Energy) are performing a broad program of investigations with relativistic nuclei. A great deal of data, obtained in an exposure of large streamer chamber to helium nuclei of 17 GeV, are being processed. A program of work, devoted to the investigation of deuterons and monochromatic neutrons in the liquid hydrogen chamber, is being completed. Figure 11 shows the b_{r1} distribution for the reaction of deuteron fragmentation obtained from paper^{/13/} submitted to this Conference. This paper is the most complete investigation of the $d + p \rightarrow ppn$ reaction in a high energy region and has many other aspects except for that noted above.

As one can see from fig. 11, the data on this exclusive reaction agree with our suggestion to classify the interactions of relativistic nuclei. In the small vicinity of b_{r1} near zero the pole approximation ("Fermi motion") describes this reaction adequately, but at large b_{r1} the regularities of the type of eq. (14) appeared.

B. Slowinski presented to this Conference results on the angular and energy distributions of protons produced in the reaction $\pi^+ + \text{Xe}$ which are relevant to our theme. His group is now preparing the exposure of a xenon bubble chamber to beams of relativistic nuclei. The 2 m propane bubble chamber is ready for an exposure to relativistic nuclei with energy up to 5 GeV/nucleon.

Track chambers will give us a large amount of data on fragmentation.

Main regularities of the nuclei fragmentation indicated by the authors of paper^{/12/} are the following:

1. The fragmentation cross sections are factorized $\sigma_{f,II}^I = C_I \cdot C_{II}$, i.e., each factor being dependent only on the properties of nuclei I or II.

2. The mean velocities of fragments are equal to the velocity of bombarding nuclei (fig. 12).

3. The momentum distributions of fragments are the same in the rest frame of fragmenting nuclei. The longitudinal momentum distributions coincide with the transverse momentum distributions and may be described by the Gaussian

$$N = a \exp \left[-p^2 / 2\phi^2 \right] \quad \text{with } \phi \simeq m_{II} = 140 \text{ MeV.}$$

The last fact is stressed by the authors of ref.^{/12/}. It is easy to explain^{/2/} these regularities by means of a usual pole approximation (see eqs. (10) and (11)).

The analysis is simplified by the application of one parameter b_{r1} instead of longitudinal and transverse momenta.

In the rest frame of nucleus I $b_{r1} \propto P_1^2 / 2m_1$. The parameter b_{r1} is expressed in terms of rapidities as follows:

$$\frac{2}{m_1} \ell_{T1} = 2 \left[\frac{m_{T1}}{m_1} \operatorname{ch}(y_T - y_1) - 1 \right] \approx (y_T - y_1)^2.$$

From this model we have the following conclusions^{/2/}:

1. To explain the factorization of the cross sections^{/12/}, it is unnecessary to use the Regge model (spin effects are neglected).

2. The equality of the mean velocities of fragments is also a consequence of the validity of the pole approximation.

3. The momentum distribution has a sharp maximum at

$$\frac{m_i}{p_i} = \frac{m_{T1}}{p_{T1}} \quad \text{or at} \quad p_i = m_i \frac{p_{T1}}{m_{T1}} = m_i \cdot (\text{const}) \quad (16)$$

in agreement with fig. 12.

4. The longitudinal and transverse momentum distributions coincide in the rest frame of fragmenting nucleus.

The statement of paper^{/12/} that all the peaks can be described by one Gauss distribution with $m_{T1} = 140$ MeV follows from the lack of accuracy of the experiment. The coincidence of the width with m_{T1} is of an accidental nature. As it follows from the semiempirical formula for the binding energy of nuclei, the mass differences in eq.(11) for α strongly change. This should lead to an essential difference in the width of the peaks in rapidity space.

It might be well to point out the investigations on nucleus-nucleus collision using internal targets in the accelerator (elastic scattering). This technique (with the participation of the same physicists) has been developed at the Dubna synchrotron and marked the beginning of the known investigations using the supersonic jet target at the Serpukhov and Batavia accelerators. We use this installation not only for relativistic nuclear physics but also for developing the technique and in order to train physi-

cists for the experiments which we are carrying out at Batavia.

To illustrate the status of these investigations (leaders A.Bujak, V.A.Nikitin), fig. 13 shows the first experimental data on the $^4\text{He} p$ elastic scattering cross sections at the He momentum 10.8 GeV/c. The mean square radius of the He nucleus is close to its value measured in the e He scattering experiments. We have similar data for small angle d-d elastic scattering in the energy of deuterons up to 10 GeV. (fig. 14).

To demonstrate the efforts of LHE in relativistic nuclear physics, let me mention the electron ray source created by the group of E.D.Donets to produce completely stripped nuclei^{/14/}. The most difficult problem of obtaining relativistic nuclei beams is to obtain fully stripped nuclei.

Now the LHE has the ion source with an intensity of 10^{11} nitrogen nuclei per pulse. JINR publications cover all the works devoted to these investigations.

Our Laboratory has made a proposal^{/15/} for the construction of a specialized cryogenic accelerator of relativistic nuclei, Nuclotron, to obtain the beams of relativistic nuclei with an energy of up to 15-20 GeV per nucleon. The project for this accelerator is now being developed.

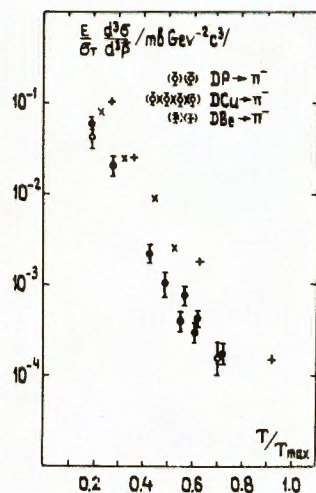


Fig. 1. Experimental data on the reactions $d + p \rightarrow \pi^-(180^\circ)$; $d + Cu \rightarrow \pi^- + \dots$ and the Berkeley results^{/10/} on the reaction $d + Be \rightarrow \pi^+ + \dots$ presented in the relativistic invariant form. Berkeley distributions do not fall as steeply as stated in^{/10/} and seem to fit. eq.(14).

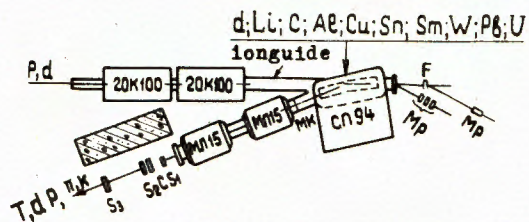


Fig. 2. Experimental layout.

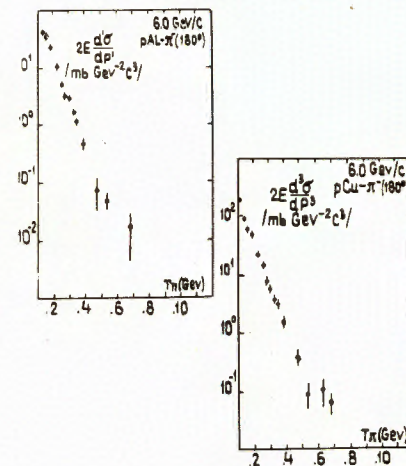


Fig. 3. Experimental data on the reaction $p + Al \rightarrow \pi^-(180^\circ)$ for primary protons with momentum 6 GeV/c. Experimental data on the reaction $p + Cu \rightarrow \pi^-(180^\circ)$ for primary protons with momentum 6 GeV/c.

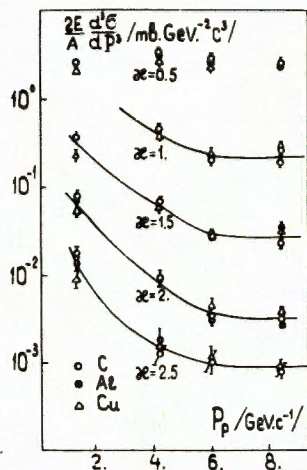


Fig. 4. The invariant cross section divided by atomic weight as a function of proton momentum P_p (\approx invariant variable ν) for various $\alpha = T/T_{\max}$ (\approx invariant variable $b_{21}/b_{11}^{m_{\pi^+}}$). Large deviations from scaling behaviour at $P_p < 2$ GeV/c are not well established. Data at this momentum were obtained by extrapolation to $\theta = 180^\circ$ of data from the literature, but not measured by us.

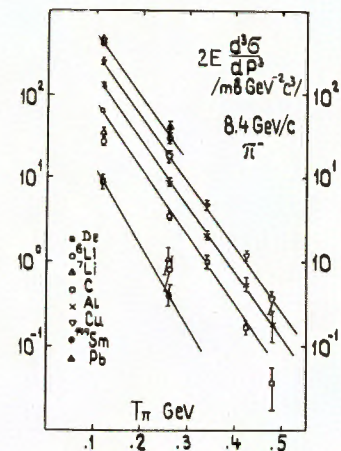


Fig. 5. Experimental data on the reaction $d + A \rightarrow \pi^-(180^\circ)$ for primary momentum $P_d = 8.4$ GeV/c.

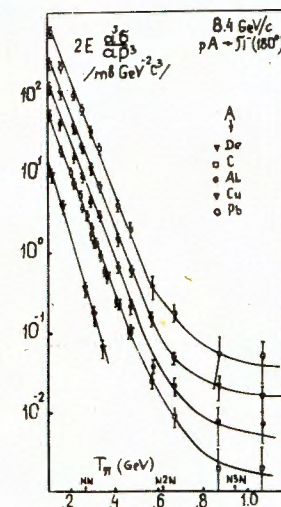


Fig. 6. Experimental data on the reactions $p + A \rightarrow \pi^-(180^\circ)$ for primary protons with momentum $P_p = 8.4$ GeV/c.

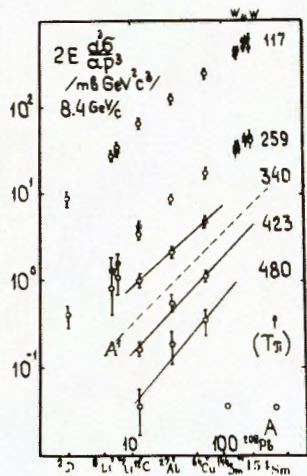


Fig. 7. f_I^I as a function of the atomic number for various energies of negative pions (T_{π^-}) for the reactions with deuterons $d + A \rightarrow \pi^-(180^\circ)$. The dashed curve is an A_I^- -reference line.

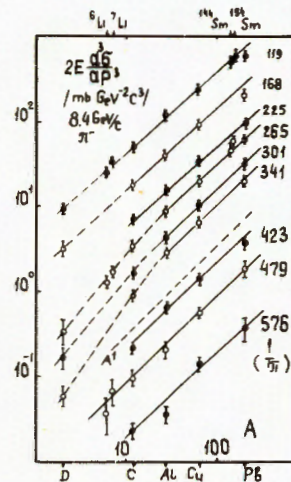


Fig. 8. f as a function of the atomic number for various energies T_{π^-} of π^- for the reactions with protons $p + A \rightarrow \pi^-(180^\circ)$. The dashed curves are A_I^- reference lines.

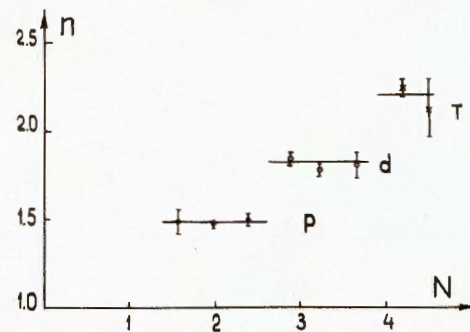


Fig. 9. The exponent n defined as $f \sim A^n$ for the reactions $p + A_I \rightarrow \left\{ \begin{matrix} p \\ d \\ T \end{matrix} \right\} (180^\circ)$ for different N -cumulative number (effective number of nucleons from A_I taking part in collision).

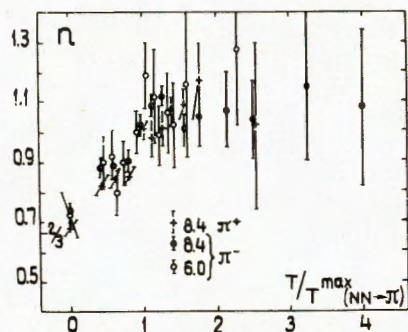


Fig. 10. The exponent n defined as $f \sim A_I^n$ for the reactions $p + A_I \rightarrow \left\{ \begin{matrix} \pi^+ \\ \pi^- \end{matrix} \right\}$ (180°) plotted versus kinetic energy T of pion. Normalization T^{\max} ($pp \rightarrow \bar{H} \dots$) is the maximum kinetic energy of pions in the reaction $p + p \rightarrow \pi$ (180°).

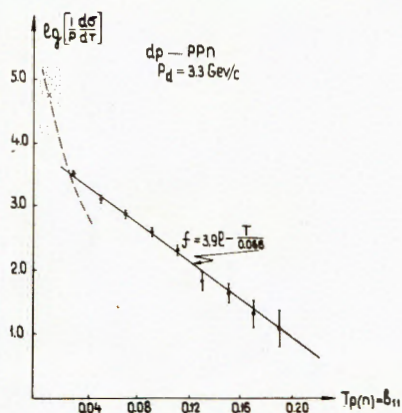


Fig. 11. Invariant cross section for the reaction $d + p \rightarrow ppn$ (hydrogen bubble chamber results^{/13/}) for protons and neutrons emitted in backward hemisphere versus invariant parameter b_{I1} .

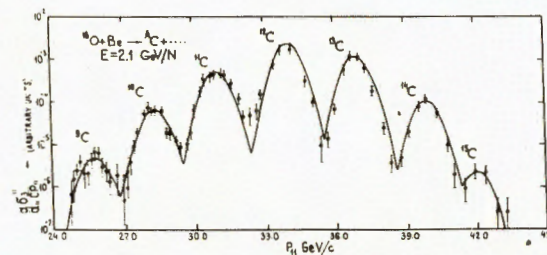


Fig. 12. Momentum distributions of various carbon fragments of the ^{16}O projectile as determined in the original measurement of Heckman et al.

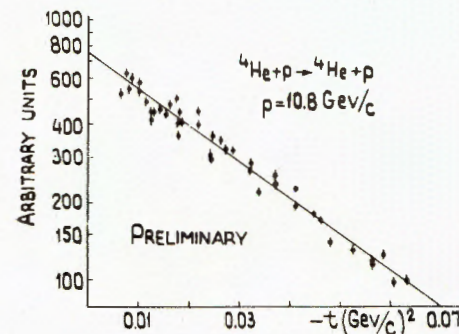


Fig. 13. Elastic differential cross section for $^4\text{He} - p$ scattering at $P_{\text{He}} = 10.8 \text{ GeV/c}$.

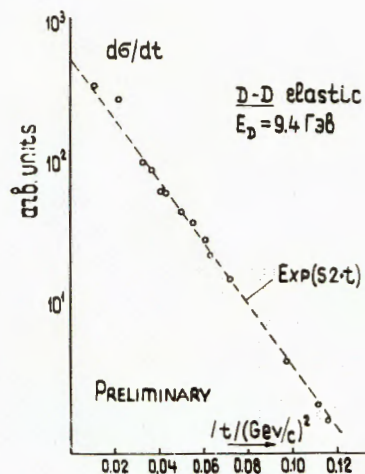


Fig. 14. Elastic differential cross section for d-d scattering at $E_d = 9.4$ GeV.

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