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S. Goloskokov

# ANOMALOUS MAGNETIC MOMENT OF NUCLEONS

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The traditional method of studying the electromagnetic structure of the nucleon are the dispersion relations in the photon mass[1]. At the present time however the great interest is excited to the investigation of the sidewise dispersion relations (Bincer[2]).

This method has been used by Ademollo et al.[3] in order to calculate the numerical value of the anomalous magnetic moment of the nucleon $\mu'$.

The anomalous magnetic moment in this method is expressed in terms of the pion-nucleon photoproduction amplitudes and the pion-nucleon vertex which are connected with the length of the $\pi N$ scattering and the parameters of the 33 resonance in the low-energy approximation. As is shown in the work of Serebryakov and Shirkov[4] these quantities in the dispersion theory depend explicitly on the nuclear anomalous magnetic moment $\mu'_N$ which is connected with the $\rho NN$ interaction.

In the present paper we present a theoretical calculation for the isovector part of the anomalous magnetic moment $\mu'^{V}$, based on the Bincer sidewise dispersion relations. We obtain the anomalous magnetic moment $\mu'^{v}$ as a function of the nuclear moment $\mu'_N$ .

The Sakurai universality principle ($\mu'^{v}=\mu'_N$) allows us to determine in unique way the anomalous magnetic moment of the nucleon which is found equal to:

$$\mu'^{v} \approx 1{,}58.$$

Let us consider the electromagnetic NN vertex $\Gamma_\mu(p_2, p)$ (Fig. 1).

In this vertex, only the ingoing nucleon is off the mass shell: $p^2 = W^2$ , $p_2^2 = m^2$ , $k^2 = 0$ . It can be shown /2/ that following projection operator $\hat{B}^{\mu(s,v)}(W)$ exists:

$$e\bar{u}(p_2)\Gamma_\mu(p_2,p)\hat{B}^{\mu(s,v)}(W)\,u(p_2) = -\frac{e}{2m}F_2^{(s,v)}(W), \tag{1}$$

where $F_2^{(s,v)}(W)$ are respectively the isoscalar and isovector parts of the electromagnetic form factor, which are equal to the anomalous magnetic moment on the mass shell: $F_2(m) = \mu'$ .

The electromagnetic form factor satisfies the following dispersion relation

$$F_2(W) = \frac{1}{\pi}\int_{m+\mu_\pi}^{\infty} dW'\left[\frac{\operatorname{Im}F_2(W')}{W'-W} + \frac{\operatorname{Im}F_2(-W')}{W'+W}\right] \tag{2}$$

the absorptive part of $F_2(W)$ is calculated with the help of the graph (Fig. 2).

In a barycentric system ($\vec{p}\,' = 0$) the dispersion relation (2) reads[3]:

$$\mu'^{\,s,v} = \frac{g}{\pi} \int_{m+\mu_\pi}^{\infty} \frac{dW}{W} \frac{|\vec{p}_2|}{|\vec{p}_1|} \left\{ \left(\frac{E_2 - m}{E_1 + m}\right)^{1/2} \frac{K^*(W)}{W-m} (W+m) M_{1-}^{s,v}(W) + \right.$$

$$\left. + \left(\frac{E_2 + m}{E_1 - m}\right)^{1/2} \frac{K^*(-W)}{W+m} (W-m) E_{0+}^{s,v}(W) \right\} . \qquad (3)$$

Here $E_{1,2} = \sqrt{\vec{p}_{1,2}^{\,2} + m^2}$ , $K(W)$ is the $\pi NN$ vertex part, normalized to the unity on the mass shell. In this case $g^2/4\pi = 14{,}6$ , $M_{1-}^{s,v}(W)$ and $E_{0+}^{s,v}(W)$ are the photoproduction amplitudes connected with the CGLN amplitudes[5]

$$A^s = 3A^{(0)} , \quad A^v = A^{(+)} + 2A^{(-)}$$

$A^{(\pm 0)}$ are the isotopic CGLN amplitudes:

$$A^a = A^{(+)} \frac{1}{2} \{ \tau^a, \tau^3 \} + A^{(-)} \frac{1}{2} [ \tau^a, \tau^3 ] + A^{(0)} \tau^a$$

We use, in what follows, the phases of the $\pi N$ scattering, which are determined in the scattering-length approximation:

$$\delta(W) = \delta_p(W) = \operatorname{arc\,tg} a_1 q^3$$

$$\delta(-W) = \delta_s(W) = \operatorname{arc\,tg} a_0 q \qquad (5)$$

where $a_0$ and $a_1$ are the $S_{11}$ and $P_{11}$ scattering lengths respectively.

In this approximation $K(\pm W)$ are [6]

$$K(W) = \frac{c}{1 - i a_1 q^3}, \quad K(-W) = \frac{c'}{1 - i a_0 q}. \tag{6}$$

Here $c$ and $c'$ are the normalization factors.

Let us calculate the photoproduction amplitudes from the CGLN dispersion relations taking into account the 33 reconance in the crossed channel [3].

$$M_{1+}^{3/2} = \frac{\mu^{v}}{f} \frac{k}{q^2} \sin\delta_{33}\, e^{i\delta_{33}} , \tag{7}$$

where $f$ is the $\pi NN$ coupling constant ($f^2 = 0{,}082$), $\mu^{v}$ is the isovector part of the total magnetic moment of the nucleon.

Then the equations for the photoproduction amplitudes in the static limit may be written in the form [3]:

$$\frac{M_{1-}^{(\pm 0)}(\omega)}{kq} = \frac{M_{1-}^{(\pm 0)B}(\omega)}{kq} + \begin{pmatrix}1\\0\\0\end{pmatrix} \frac{8}{9\pi} \int_{\mu_\pi}^{\infty} \frac{d\omega'}{\omega' + \omega} \frac{\operatorname{Im} M_{1+}^{3/2}(\omega')}{k'q'} + \frac{1}{\pi} \int_{\mu_\pi}^{\infty} \frac{d\omega'}{\omega' - \omega} \frac{\operatorname{Im} M_{1-}^{(\pm 0)}(\omega')}{k'q'}$$

$$\frac{E^{(\pm 0)}_{0+}(\omega)}{\omega} = \frac{E^{(\pm 0)B}_{0+}(\omega)}{\omega} + \begin{pmatrix}1\\0\\0\end{pmatrix} \frac{4}{3\pi}\int_{\mu_\pi}^{\infty} \frac{d\omega'}{k'q'} \operatorname{Im} M^{3/2}_{1+}(\omega') + \frac{1}{\pi}\int_{\mu_\pi}^{\infty} \frac{d\omega'}{\omega'-\omega} \frac{\operatorname{Im} E^{(\pm 0)}_{0+}(\omega')}{\omega'}, \tag{8}$$

where $\omega$ is the static variable: $\omega = W - m$. The absorptive parts of the amplitudes are calculated in the two-particle approximation (Fig. 3).

The contributions of the $\Delta_{33}$ and the Born terms to the electric and magnetic parts of the photoproduction amplitudes $A = A_e + A_m$ ( $A_m$ contains the total magnetic moment of the nucleon) are the following /2/: for the multipoles $M^{(+)}_{1-m}(\omega)$ and $E^{(+)}_{0+m}(\omega)$:

$$M^{(+)}_{1-m}(\omega) = e^{i\delta(W)} \cos\delta(W) \left[ M^{(+)B}_{1-m}(\omega) + \frac{2}{3}\frac{\mu^{v}}{f} Z'_{33}(\omega)\frac{kq}{\omega} \right]$$

$$E^{(+)}_{0+m}(\omega) = e^{i\delta(-W)} \cos\delta(-W) \left[ E^{(+)B}_{0+m} + \frac{\mu^{v}}{f} Z_{33} \cdot \omega \right] \tag{9}$$

$\delta(\pm W)$ are the $\pi N$ scattering phases (5), $Z_{33}$ and $Z'_{33}(\omega)$ contain the contribution of the 33 resonance

$$Z_{33} = \frac{4}{3\pi}\int d\omega' \frac{\sin^2\delta_{33}(\omega')}{q'^3}$$

$$Z'_{33}(\omega) = \frac{4}{3\pi}\int d\omega' \frac{\omega}{\omega'+\omega}\frac{\sin^2\delta_{33}(\omega')}{q'^3} \tag{10}$$

the $\Delta_{33}$ does not contribute to the other multipoles

$$M^{(\alpha)}_{1-(e,m)} = e^{i\delta(W)} \cos\delta(W) M^{(\alpha)B}_{1-(e,m)}$$

$$E^{(\alpha)}_{0+(e,m)} \; e^{i\delta(-W)} \cos\delta(-W)\, E^{(\alpha)B}_{0+(e,m)} . \tag{11}$$

Taking into account the contribution of the 33 resonance in $\delta$-approximation

$$\sin^2\delta_{33} = \pi\Gamma_{33}\, q^3\, \delta(\omega_{33} - \omega) \tag{12}$$

we get that $Z_{33}$ and $Z'_{33}(\omega)$ depend on the width and the mass of the $\Delta_{33}$ . The photoproduction amplitudes and the $\pi NN$ vertex $(K(\omega))$ also depend on the $\pi N$ scattering length.

Thus the isovector part of the anomalous magnetic moment depends on the four parameters

$$\mu'^{V} = \Phi\, (a_0, a_1, \omega_{33}, \Gamma_{33}) . \tag{13}$$

Let us determine the s-scattering length from experiment and the other quantities from the analysis of the equation for the p-waves of the $\pi N$ scattering/4/

$$p^{(\pm)} = p^{(\pm)B} + \frac{1}{\pi}\begin{pmatrix} 1 \\ \omega \end{pmatrix} \int_0^\infty \frac{\operatorname{Im} p^{(\pm)}(\nu')}{\nu' - \nu} \begin{pmatrix} 1 \\ \frac{1}{\omega'} \end{pmatrix} d\nu'$$

$$\beta^{(\pm)} = \beta^{(\pm)B} + \frac{1}{\pi}\begin{pmatrix} \omega \\ 1 \end{pmatrix} \int_0^\infty \frac{\operatorname{Im} \beta^{(\pm)}(\nu')}{\nu' - \nu} \begin{pmatrix} \frac{1}{\omega'} \\ 1 \end{pmatrix} d\nu' \tag{14}$$

here $\nu$ is the square of the $\pi$ -meson momentum.

The Born terms of the both equations depend on the nuclea anomalous magnetic moment connected with the $\rho NN$ vertex.

From the asymptotic and the threshold conditions for equations (14) we can determine the parameters of the 33 resonance[4]

$$\Gamma_{33} = 0{,}068, \quad \omega^*_{33} = 0{,}465\,(2\mu'_N + 1)\ . \tag{15}$$

The scattering length $a_1$ , as determined from the equatior for the $h_1 \equiv h_{11}$ $\pi N$ scattering amplitude[4] is

$$h_1(\omega) = \Phi_1(\omega) + \frac{1}{\pi}\int_{\mu_\pi}^\infty d\omega' \left[ \frac{\operatorname{Im} h_1(\omega')}{\omega' - \omega} + A_{1k} \frac{\operatorname{Im} h_k(\omega')}{\omega' + \omega} \right] \tag{16}$$

is equal to

$$a_1 = -0{,}202 + 0{,}006\,(2\mu'_N + 1) + 0{,}157/(\omega_{33} + \mu_\pi)\,. \tag{17}$$

The first two terms in eq. (17) arise from the amplitude $\Phi$ and the last term from taking into account of the $\Delta_{33}$ in the crossing channel.

Hence, the parameters entering eq. (13) depend explicitly on the nuclear anomalous magnetic moment of the nucleon. Therefore the anomalous magnetic moment $\mu'^{v}$ is some function of the nuclear moment $\mu'_N$.

$$\mu'^{v} = \Phi(\mu'_N)\,. \tag{18}$$

The Sakurai universality principle [7] requires the equality $\mu'^{v} = \mu'_N$.

Thus the anomalous magnetic moment of the nucleon is determined in this model as an interception point of the function $\mu'^{v} = \Phi(\mu'_N)$ and the straight line $\mu'^{v} = \mu'_N$.

Let us write the equation for the $\mu'^{v}$ in the static limit, taking into account the $1/W$ correction (see eq. 3).

$$\mu'^{v} = \frac{g}{\pi m} \int_{\mu_\pi}^{\infty} \frac{d\omega}{1+\frac{\omega}{m}} \frac{q}{\omega} \left[\frac{q}{\omega} K^*(\omega) M^{v}_{1-}(\omega) + K^*(-\omega) E^{v}_{0+}(\omega)\right]. \tag{19}$$

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The Born terms for the photoproduction amplitudes are taken in the form.

$$M^{(\pm)B}_{1-m} = -\frac{kq}{3\omega} f \begin{pmatrix} 4\mu^{\mathrm{v}} \\ 2\mu^{\mathrm{v}} \end{pmatrix} ; \quad M^{(\pm)B}_{1-e} = \frac{2}{3} kqef \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right) F_m$$

$$E^{(\pm)B}_{0+m} = -f\omega \begin{pmatrix} 2\mu^{\mathrm{v}} \\ 0 \end{pmatrix} ; \quad E^{(\pm)B}_{0+e} = ef \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right) F_e \quad , \tag{20}$$

where $\mu^{\mathrm{v}}$ is a total magnetic moment of the nucleon,

$$F_m = \frac{3}{4q^2}\left(1+\frac{1-v^2}{2v}\ln\left|\frac{1-v}{1+v}\right|\right); \; F_e = 1-\frac{1}{2}\left(1+\frac{1-v^2}{2v}\ln\left|\frac{1-v}{1+v}\right|\right); \; v=\frac{q}{\omega}$$

In this case all the integrals in eq. (19) are finite and the cutoff is not needed.

Separating from the integrals on the right hand side of eq. (19) the electric and magnetic terms we have:

$$\mu'^{\mathrm{v}} = L_e(\mu'_N) + \mu^{\mathrm{v}} L_m(\mu'_N) \; . \tag{21}$$

From eq. (21) follows:

$$\mu'^{\mathrm{v}} = \frac{L_e(\mu'_N) + \frac{1}{2} L_m(\mu'_N)}{1 - L_m(\mu'_N)} = \Phi(\mu'_N) \; . \tag{22}$$

In Fig. 4 we represent the function $\Phi(\mu_N')$ which is approximately constant in a large region of the variable.

A sharp dip of $\Phi(\mu_N')$ is connected with the divergence of the integrals of $M^{\triangledown}_{1-m}$ which is caused by the zero of $a_1(\mu_N')$ at the point $\mu_N' \approx 16$ (see Fig. 5).

The anomalous magnetic moment is

$$\mu'^{\triangledown} = 1{,}58 \ .$$

The functions $L_e$ and $L_m$ at this point are equal to:

$$L_e = 13{,}52 \ ; \ L_m = -5{,}75 \quad .$$

Note the connection of this work with the paper of Serebryakov and Shirkov /4/ where it was shown that the threshold and the asymptotic conditions for the $\pi N$ dispersion relations allow to exhibit some relations between the theoretical parameters. However the anomalous magnetic moment of the nucleon can not be determined in such a way. In the present paper it is determined in a unique way, reducing the number of the free parameters. Because of the large angle interception of the function $\Phi(\mu_N')$ with the straight line the intercept point is clearly seen.

The uniqueness of the determination of the anomalous magnetic moment of the nucleon is a main result of the present paper.

The obtained value of the anomalous magnetic moment is in good agreement with the experimental data.

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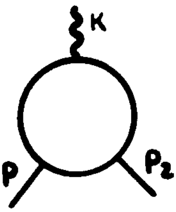


Fig. 1

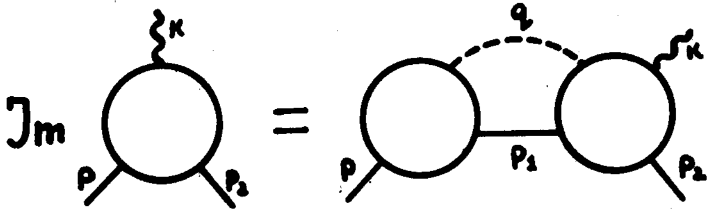


Fig.2

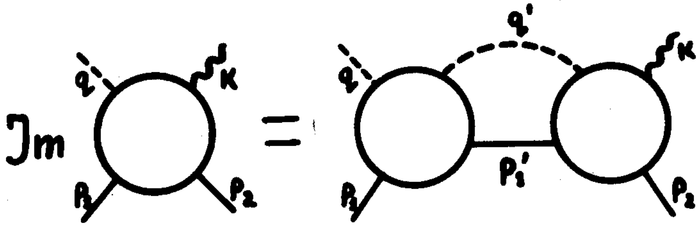


Fig.3

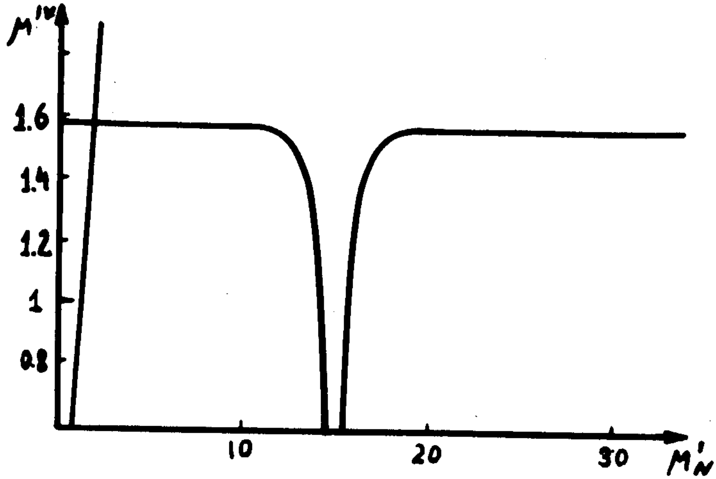


Fig.4

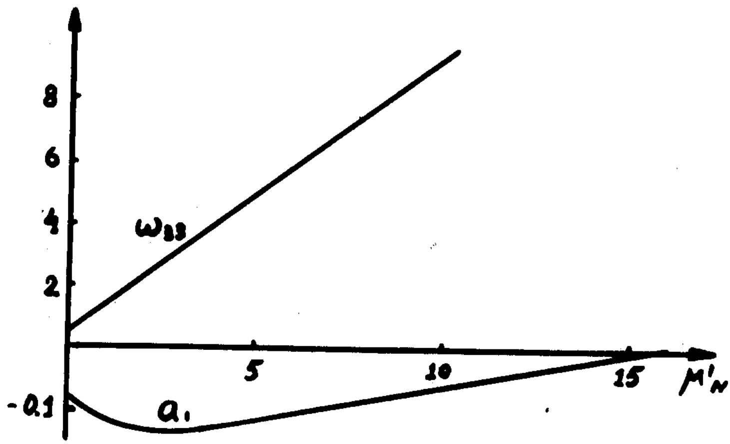


Fig.5