ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2 - 5198

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

M.K. Volkov

ANALYTICAL STRUCTURE OF THE SUPER-PROPAGATOR IN THE COUPLING CONSTANT

1970

E2 - 5198

M.K. Volkov

ANALYTICAL STRUCTURE OF THE SUPER-PROPAGATOR IN THE COUPLING CONSTANT

Submitted to "Теоретическая и математическая физика"

1. Introduction

The problem of the construction of the two-point Green functions in the theories with rapidly increasing spectral functions was investigated by many authors in a lot of works during last fifteen years [1-5]. Further for brevity we shall call these functions the super-propagators, following Salam's terminology [1]. The first and one of the interesting works in that domain was the S. Okubo's paper [2]. It was as early as 1954 year. In the following years many authors often returned to different aspects of this problem [3-5]. In their papers a few methods of the construction of the super-propagators have been suggested, which satisfy the conditions of causality and unitarity of the S -matrix. So far as the special work of the author of this paper was dedicated to the review of all these methods, we shall not discuss it, but shall study a problem closely connected with a subsequent development of our method.

In a paper [4] exact expressions for the super-propagators of the scalar particles with zero rest mass have been obtained. Further the problem of generalization of these calculations on the massive case naturally arose. It is quite obvious that all the calculations in the massive case are very complicated. There is no hope to obtain here the exact results contrary to the massless case. However, we can obtain approximate expressions for the super-propagators which can be successfully used in certain domains of the values of the momentum. It was made in paper /5/, for instance, where one obtained an approximate expression for the super-propagator well describing its behaviour for the large values of the momentum as well as location of all its thresholds. Yet the analytical structure of the super-propagator in the coupling constant g is reproduced inexactly in that approximation.

The calculations performed in this paper clearly show that in the massive and the massless cases the super-propagators essentially differ from each other not only by their dependence upon the momentum of particles (if interaction is nonlocal, even asymptotics for the large momenta of such super-propagators are strongly different), but also by the analytical structure in the coupling constant g. Really, if the super-propagator in the case of the massless particles has only a logarithmic branch point with respect to g for $g=0$ in the momentum space, in the massive case it has much more complicated nonanalyticity in the coupling constant.

In addition to the elucidation of the general analytical structure of the super-propagator in the coupling constant for the massive particles, an expression obtained for the super-propagators, can be used for describing it for small values of the momentum and the coupling constant g.

2. Super-Propagator of Massive Scalar Particles

In the configuration space the super-propagator of scalar particles can be written in the form [4,5]

$$\Phi(x) = i \sum_{1}^{\infty} a(n) \left[ig^2 \Delta^c(x) \right]^n , \tag{1}$$

where

$$\Delta^c_m(x) = \frac{1}{(2\pi)^4} \int d^4p \frac{e^{ipx}}{m^2 - p^2 - i\epsilon} , \tag{2}$$

g is the coupling constant and $a(n)$ is the coefficient characterizing some concrete interaction of scalar particles.

Using the results obtained in paper [5] we go over to the momentum space. Here we have for the super-propagator

$$\tilde{\tilde{\Phi}}(p) = \frac{(2\pi)^2 \kappa\, a(1)}{p^2 - m^2 + i\epsilon} + \bar{\Phi}(p) , \qquad \kappa = \left(\frac{g}{2\pi} \right)^2 . \tag{3}$$

In the region $p^2 < 0$ $\bar{\Phi}(p)$ can be represented in the form of the integral

$$\bar{\Phi}(p) = (i)^{-1} 2(\pi\kappa)^2 \frac{m}{|p|} \int_{-a+i\infty}^{-a-i\infty} \frac{dz}{\sin \pi z} (\kappa m^2)^z a(z+2) \int_0^{\infty} dr\, r^{-z} (K_1(r))^{z+2} J_1\left(\frac{|p|}{m} r \right) , \tag{4}$$

where $K_1(r)$ and $J_1(\frac{|p|}{m} r)$ are Bessel functions, $|p| = \sqrt{-p^2}$ and $0 < a < 1$.

The integral (4) cannot be taken exactly. In paper [5] one suggested a procedure of an approximate calculation of this integral, which perfectly described the asymptotical behaviour of the super-propagator for large values of the momentum. Now we show how one can get the expression for the $\tilde{\tilde{\Phi}}(p)$, which correctly reproduces the behaviour of the super-propagator for small values of the momentum (near the values of the first thresholds $\sim 4m^2$, $9m^2$, $16m^2$...) and the coupling constant κ. In other words, we want to construct some modification of the usual perturbation theory with expansion in constant κ, when words " n-order in κ ", mean that our expression has the form $\kappa^n f(\kappa, p^2)$, where function $f(\kappa, p^2)$ can depend on $\ln \kappa$, besides p^2.

Calculating " n-order in κ " we shall use the following approximate expression for the super-propagator

$$\Phi_n(p) = (i)^{-1} 2(\pi\kappa)^2 \frac{m}{|p|} \int_{-a+i\infty}^{-a-i\infty} \frac{dz}{\sin \pi z} (\kappa m^2)^z a(z+2) \int_0^\infty dr\, r^{-2(z+1)+n} (K_1(r))^n J_1\left(\frac{|p|}{m} r\right). \quad (5)$$

In this expression " n-order in κ " is most near " n-order in κ " of the exact formula (4). Other "orders in κ " are deviated from similar orders of the formula (4) the more, the further they are away from value n.

After these general comments we are engaged in calculating the "second", "third" and "fourth orders in κ ".

3. "The Lowest Orders in κ "

Let us calculate "the second order in κ " of the super propagator $\tilde{\tilde{\Phi}}(p)$. For that we rewrite formula (5) in the form

$$\Phi_2(p) = (i)^{-1} 2(\pi\kappa)^2 \frac{m}{|p|} \int_{-a+i\infty}^{-a-i\infty} \frac{dz}{\sin \pi z} (\kappa m^2)^z a(z+2)\, b_2\left(\frac{|p|}{m}, z\right), \quad (6)$$

where

$$b_2\left(\frac{|p|}{m}, z\right) = \int_0^\infty dr\, r^{-2z} K_1^2(r)\, J_1\left(\frac{|p|}{m} r\right) =$$

$$= \frac{\sqrt{\pi}}{8} \frac{|p|}{m} \sum_0^\infty \left(\frac{p^2}{4m^2}\right)^k \frac{\Gamma(k-z)\Gamma(k-z+1)\Gamma(k-z+2)}{k!(k+1)!\ \Gamma(k-z+3/2)} . \qquad (7)$$

and $\Gamma(z)$ is the gamma function. We calculated the integral (7), using the representation of the Bessel function $J_1\left(\frac{|p|}{m} r\right)$ by a power series. With the help of these expressions we can write "the second order in κ" in the form

$$\bar{\Phi}_2^{(2)}(p) = \frac{\sqrt{\pi}}{4i} (\pi\kappa)^2 \sum_0^\infty \frac{\left(\frac{p^2}{4m^2}\right)^k}{k!(k+1)!} h_k , \qquad (8)$$

where

$$h_0 = -\pi \int \frac{dz}{\sin^2 \pi z} (\kappa m^2)^z a(z+2) \frac{\Gamma(1-z)\Gamma(2-z)}{\Gamma(1+z)\Gamma(3/2-z)} =$$

$$(9')$$

$$= -i \frac{4}{\sqrt{\pi}} a(2) \left[\ln \frac{\kappa m^2}{4} + (\ln a(\nu))' /_{\nu=2} + 2C + 1 \right],$$

$$h_{k \atop (k\neq 0)} = \int \frac{dz}{\sin \pi z} (\kappa\, m^2)^z\, a(z+2) \frac{\Gamma(k-z)\Gamma(k-z+1)\Gamma(k-z+2)}{\Gamma(k-z+3/2)} =$$

$$= i\, 2a(2) \frac{\Gamma(k)\,\Gamma(k+1)\,\Gamma(k+2)}{\Gamma(k+3/2)} . \qquad (9'')$$

Here C is the Euler constant and the integrals are the residues at the zero. Inserting (9') and (9'') into (8) and summing up the series (see Appensix) we finally get the expression for "the second order in κ " of the super-propagator ($p^2 < 0$) :

$$\overset{=(2)}{\Phi}(p) = \overset{-(2)}{\Phi}(p) = -(\pi\kappa)^2 a(2) \Big\{ \sqrt{1-\frac{4m^2}{p^2}}\ \ell n \frac{\sqrt{1-\frac{4m^2}{p^2}}+1}{\sqrt{1-\frac{4m^2}{p^2}}+1} + \ell n \frac{\kappa m^2}{4} + \qquad (10)$$

$$+ (\ell n\ a(\nu))'\big|_{\nu=2} + 2C - 1 \Big\} .$$

The formula (10) differs from the well-known expression for the scalar loop only by that, the former contains a term with evidently nonanalytical dependence on the coupling constant $\ell n \frac{\kappa m^2}{4}$.

Let us consider "the third order in κ ". The function $\Phi_3(p)$ used for the calculation of "the third order in κ " differs from the foregoing function $\Phi_2(p)$ by that its integrand contains $b_3\left(\frac{|p|}{m}, z\right)$ instead of $b_2\left(\frac{|p|}{m}, z\right)$. The function $b_3\left(\frac{|p|}{m}, z\right)$ is equal to

$$b_3\left(\frac{|p|}{m}, z\right) = \int_0^\infty dr\, r^{-2z+1} K_1^3(r)\, J_1\left(\frac{|p|}{m} r\right) . \qquad (11)$$

Using again the representation of the Bessel function $J_1\left(\frac{|p|}{m} r\right)$ by the series and one of the functions $K_1(r)$ by the integral

$$K_1(r) = \frac{\pi}{4i} \int_{-\beta+i\infty}^{-\beta-i\infty} dv \frac{\left(\frac{r}{2}\right)^{2v-1}}{\sin^2 \pi v\; \Gamma(v)\Gamma(v+1)} , \qquad (12)$$

we can take the integral over r

$$b_3\left(\frac{|p|}{m}, z\right) = -i\frac{\pi^{3/2}}{16} \frac{|p|}{m} \sum_0^\infty \frac{\left(\frac{p^2}{4m^2}\right)^k}{k!(k+1)!} \int_{-\beta+i\infty}^{-\beta-i\infty} dv \frac{\Gamma(k+v-z)\Gamma(k+v-z+1)\Gamma(k+v-z+2)}{\sin^2 \pi v\, 4^v\, \Gamma(v)\Gamma(v+1)\Gamma(k+v-z+3/2)} .$$

$$(0 < \beta < 1)$$

Withe the help of this formula we come to the following expression for the $\bar{\Phi}_3(p)$

$$\bar{\Phi}_3(p) = \frac{\pi^{7/2}}{8} \kappa^2 \sum_0^\infty \frac{\left(\frac{p^2}{4m}\right)^k}{k!(k+1)!} \int_{-\alpha-i\infty}^{-\alpha+i\infty} dz \frac{(\kappa m^2)^z}{\sin \pi z} a(z+2) \int_{-\beta+i\infty}^{-\beta-i\infty} dv \frac{\Gamma(k+v-z)\,\Gamma(k+v-z+1)\,\Gamma(k+v-z+2)}{\sin^2 \pi v\; 4^v\, \Gamma(v)\Gamma(v+1)\Gamma(k+v-z+3/2)} .$$

Taking into account that $\beta < \alpha$ we obtain for "the third order in κ" of the super-propagator

$$\tilde{\tilde{\Phi}}^{(3)}(p) \cong \bar{\Phi}_3^{(3)}(p) = \frac{3}{8}\pi^2 a(3)\, m^2 \kappa^3 \Big\{ \Big(\ell n \frac{\kappa m^2}{4}\Big)^2 +$$

$$+2\Big(\ell n \frac{\kappa m^2}{4}\Big)\Big[(\ell n\ a(\nu))'\Big|_{\nu=3} + 2C - 1 + \frac{p^2}{6m^2}\Big] + f_3(p^2)\Big\}, \tag{15}$$

where

$$f_3(p^2) = \frac{i2\pi^{3/2}}{3}\sum_0^{\infty}\frac{\left(\frac{p^2}{4m^2}\right)^k}{k!(k+1)!}\int_{-\beta+i\infty}^{-\beta-i\infty} dv\, \frac{\Gamma(k+v-1)\Gamma(k+v)\Gamma(k+v+1)}{\sin^2\pi v\, 4^v\Gamma(v)\Gamma(v+1)\Gamma(k+v+1/2)} +$$

$$+\frac{1}{3}\frac{p^2}{m^2}\Big[(\ell n\ a(\nu))'\Big|_{\nu=3} + 2C\Big] + \text{const}. \tag{16}$$

Unfortunately, $f_3(p^2)$ is not expressed in the elementary functions, what is the case for "the second order in κ". Therefore we do not sum the series in the powers of $\left(\frac{p^2}{4m^2}\right)$.

Let us also consider "the fourth order in κ", in order to observe the change of the analytical structure of the super-propagator in the coupling constant when passing from order to order. The function $b_4\left(\frac{|p|}{m}, z\right)$ has the form

$$b_4\left(\frac{|p|}{m}, z\right) = \int_0^{\infty} dr\, r^{-2z+2} K_1^4(r)\, J_1\left(\frac{|p|}{m} r\right). \tag{17}$$

Representing the two functions $K_1(r)$ by integrals and the function $J_1(\frac{|p|}{m} r)$ by a series we get for the $\bar{\Phi}_4(p)$

$$\bar{\Phi}_4(p) = i\frac{\pi^{9/2}}{16}\kappa^2 \sum_0^\infty \frac{\left(\frac{p^2}{4m^2}\right)^k}{k!\,(k+1)!} \int_{-a+i\infty}^{-a-i\infty} dz\, \frac{(\kappa m^2)^z}{\sin\pi z} a(z+2) \times$$

$$\times \int\int_{-\beta+i\infty}^{-\beta-i\infty} dv\, du\, \frac{4^{-v-u}\Gamma(k-z+v+u)\,\Gamma(k-z+v+u+1)\,\Gamma(k-z+u+2)}{\sin^2\pi v\,\sin^2\pi u\,\Gamma(v)\Gamma(v+1)\Gamma(u)\Gamma(u+1)\Gamma(k-z+v+u+3/2)} \tag{18}$$

Here the inequality $2\beta < a$ takes place. From (18) we obtain the following expression for "the fourth order in κ " of the super-propagator

$$\tilde{\tilde{\Phi}}^{(4)}(p) = \bar{\Phi}_4^{(4)}(p) = \frac{(-3)}{8}\pi^2 a(4)(m\kappa)^4\{(\ln\frac{\kappa m^2}{4})^3 + (\ln\frac{\kappa m^2}{4})^2[(\ln a(\nu))'|_{\nu=4} + 2C - \frac{5}{6} + \frac{p^2}{6m^2}] +$$

$$+ (\ln\frac{\kappa m^2}{4})[(\ln a(\gamma))''|_{\nu=4} + ((\ln a(\nu))'|_{\nu=4} + 2C - \frac{5}{6} + \frac{p^2}{6m^2})^2 - \frac{p^2}{18m^2} - \frac{1}{2}(\frac{p^2}{6m^2})^2 - \pi^2 - \frac{19}{36}] + \tag{19}$$

$$+ f_4(p^2)\} .$$

The function $f_4(p^2)$ does not depend upon the coupling constant

Basing on this investigation of the super-propagator in "the first four orders in κ " we can conclude that the super propagator has, in general, the form of the following series

$$\tilde{\tilde{\Phi}}(p) = \frac{\kappa}{m^2} \sum_0^\infty (\kappa m^2)^n a(n+1) [f_{n+1}(p^2) + (\ell n\, \kappa m^2)\, \phi^{(n)}_{n-1}\left(\frac{p^2}{m^2}\right) +$$

$$+ (\ell n\, \kappa m^2)^2 \phi^{(n)}_{n-2}\left(\frac{p^2}{m^2}\right) + \ldots + (\ell n\, \kappa m^2)^n\, \phi^{(n)}_0\left(\frac{p^2}{m^2}\right)] , \tag{20}$$

where $\phi^{(n)}_k\left(\frac{p^2}{m^2}\right)$ is the polynomial of the k -power in $\left(\frac{p^2}{m^2}\right)$ - and $f_{n+1}(p^2)$ is a function of p^2 which is independent of the coupling constant κ .

From formula (20) one can see that the nonanalyticity in the coupling constant of the massive super-propagator has a very complicated form. Here can exist even the essential sinsularity with respect to κ for $\kappa = 0$.

4. Conclusion

In the approximate formulae for the super-propagators, obtained in paper /5/, the nonanalyticity in the coupling constant has simpler form. Namely, the super-propagator had only a logarithmic branch point with respect to κ . This is explained by the fact that a very crude approximation for the function $K_1(r)$ has been used in the above-mentioned paper

$$K_1(r) \approx \frac{e^{-r}}{r} ,$$

which has permitted to describe well the super-propagator behaviour for the large values of the momentum and correctly reproduce their structure in the momentum of the particles (the position of the thresholds). However, in order to obtain a correct dependence of the super-propagator upon the coupling constant, it is necessary to take a more strict approximation for the function $K_1(r)$, which takes into account in particular the presence of the logarithmic dependence of the $K_1(r)$ on r.

Notice in conclusion that in a recent interesting paper A. Salam and J. Strathdee /1/, using our method /4/, have investigated the problem of the unitarity of the S -matrix in higher perturbation orders in the "major" coupling constant in theories with nonpolynomial Lagrangians. These authors considered both the massless and the massive cases. In connection with the latter we remark that they make on incorrect assumption about the analytical structure of the super-propagator in the coupling constant κ. Namely, their assumption, that the massive super-propagator has only a simple logarithmic branch point with respect to κ, is wrong, which follows from the calculations presented here.

Appendix

Let us consider the series

$$A(a) = \frac{\sqrt{\pi}}{2} \sum_{1}^{\infty} a^k \frac{\Gamma(k)}{\Gamma(k+3/2)} = \sum_{1}^{\infty} (4a)^k \frac{\Gamma(k)\,\Gamma(k+1)}{\Gamma(2k+2)}. \qquad (A.1)$$

Using for the integral representations

$$\Gamma(k) = \int_0^{\infty} dt\; t^{k-1}\, e^{-t} \qquad (A.II)$$

and summing up the series we get

$$A(a) = \int\int_0^{\infty} ds\ dt\ t^{-1} e^{-t-s} \frac{sh(\sqrt{4a\,ts}) - \sqrt{4a\,t\,s}}{\sqrt{4a\,t\,s}} . \qquad (A.III)$$

Let us make the replacement of the variables: $s = s$ and $\nu = \sqrt{4ts}$. Now we can easily take the integral over

$$A(a) = 2 \int_0^{\infty} d\nu\ K_1(\nu) \frac{sh(\sqrt{a}\,\nu) - \sqrt{a}\,\nu}{\sqrt{a}\,\nu} =$$

$$= 2\sqrt{a} \int_0^1 dt\,(1-t) \int_0^{\infty} d\nu\,\nu\ K_1(\nu)\ sh\,(\sqrt{a}\,t\nu) . \qquad (A.IV)$$

We can also take the integral over ν (see [7])

$$A(a) = 2\sqrt{a} \int_0^1 dt \frac{1-t}{1-at^2} [\sqrt{a}\,t + \frac{\arcsin(\sqrt{a}\,t)}{\sqrt{1-at^2}}] \qquad (A.V)$$

$$(a < 1) .$$

Calculating this last integral we definitively obtain

$$A(a) = 2\,[1 - \sqrt{\frac{1}{a} - 1}\ \mathrm{arc\,tg}\,(\frac{1}{a} - 1)^{-1/2}] \quad (0 < a < 1)$$

$$(A.VI)$$

$$A(a) = 2 - \sqrt{1 - \frac{1}{a}}\ \ln \frac{\sqrt{1-\frac{1}{a}} + 1}{\sqrt{1-\frac{1}{a}} - 1} \quad (a < 0) .$$

which has permitted to describe well the super-propagator behaviour for the large values of the momentum and correctly reproduce their structure in the momentum of the particles (the position of the thresholds). However, in order to obtain a correct dependence of the super-propagator upon the coupling constant, it is necessary to take a more strict approximation for the function $K_1(r)$, which takes into account in particular the presence of the logarithmic dependence of the $K_1(r)$ on r.

Notice in conclusion that in a recent interesting paper A. Salam and J. Strathdee /1/, using our method /4/, have investigated the problem of the unitarity of the S-matrix in higher perturbation orders in the "major" coupling constant in theories with nonpolynomial Lagrangians. These authors considered both the massless and the massive cases. In connection with the latter we remark that they make on incorrect assumption about the analytical structure of the super-propagator in the coupling constant κ. Namely, their assumption, that the massive super-propagator has only a simple logarithmic branch point with respect to κ, is wrong, which follows from the calculations presented here.

Appendix

Let us consider the series

$$A(a) = \frac{\sqrt{\pi}}{2} \sum_{1}^{\infty} a^k \frac{\Gamma(k)}{\Gamma(k+3/2)} = \sum_{1}^{\infty} (4a)^k \frac{\Gamma(k)\Gamma(k+1)}{\Gamma(2k+2)}. \qquad (A.1)$$

Using for the integral representations

$$\Gamma(k) = \int_0^\infty dt \; t^{k-1} e^{-t} \qquad (A.II)$$

and summing up the series we get

$$A(a) = \int\int_0^{\infty} ds\; dt\; t^{-1} e^{-t-s} \frac{sh(\sqrt{4a\,ts}) - \sqrt{4a\,t\,s}}{\sqrt{4\,a\,t\,s}} . \qquad \text{(A.III)}$$

Let us make the replacement of the variables: $s = s$ and $\nu = \sqrt{4ts}$. Now we can easily take the integral over

$$A(a) = 2 \int_0^{\infty} d\nu \quad K_1(\nu) \frac{sh(\sqrt{a}\,\nu) - \sqrt{a}\,\nu}{\sqrt{a}\,\nu} =$$

$$= 2\sqrt{a} \int_0^1 dt\,(1-t) \int_0^{\infty} d\nu\,\nu\; K_1(\nu)\; sh\,(\sqrt{a}\;\; t\nu\;) . \qquad \text{(A.IV)}$$

We can also take the integral over ν (see /7/ 6)

$$A(a) = 2\sqrt{a} \int_0^1 dt\; \frac{1-t}{1-at^2} \left[\sqrt{a}\;\; t + \frac{\arcsin\,(\sqrt{a}\;\; t\;)}{\sqrt{1-at^2}} \right] \qquad \text{(A.V)}$$

$$(a < 1) .$$

Calculating this last integral we definitively obtain

$$A(a) = 2\left[1 - \sqrt{\frac{1}{a} - 1}\; \mathrm{arc\,tg}\left(\frac{1}{a} - 1\right)^{-1/2} \right] \quad (0 < a < 1)$$

(A.VI)

$$A(a) = 2 - \sqrt{1 - \frac{1}{a}}\; \ell n \frac{\sqrt{1-\frac{1}{a}} + 1}{\sqrt{1-\frac{1}{a}} - 1} \quad (a < 0) .$$

References

1. A. Salam, J. Strathdee. Preprint IC/69/120, Trieste (1969).
2. S. Okubo. Progr. Thoret. Phys. (Kyoto), II, 80 (1954).
   Nuovo Cim., 19, 574 (1961).
3. R. Arnowitt and S. Deser. Phys.Rev., 100, 349 (1955).
   Г.В. Ефимов. ЖЭТФ, 44, 2107 (1963); 48, 596 (1965).
   Nuovo Cim., 32, 1046 (1964). E.S. Fradking. Nucl.Phys., 49, 624 (1963). G. Feinberg and A. Pais. Phys.Rev., 131, 2714 (1963). W. Güttinger. Fortschr. Phys., 14, 483 (1966).
   М.К. Волков. ЯФ, 6, 1100 (1967); 7, 445 (1968). Б.А. Арбузов, Н.М. Атакишиев, А.Т. Филиппов. ЯФ, 8, 365 (1968).
   H.M. Fried. Nuovo Cim., 52A, 1333 (1967). R. Delbourgo, A. Salam and J. Strathdee. Preprint ICTP, IC/69/17, Trieste (0969).
   B.E. Lee and B. Zumino. Preprint TH. 1053, CERN (1969).
   H. Lehmann and K. Pohlmeyer.
   The Propagator in a Field Theoretical Model with Exponential Coupling, Prepr. of Inst. Theor. Phys,, Hamburg, 1970.
   B.W. Keck and J.G. Taylor. Prepting University of Southampton, April, 1970.
4. M.K. Volkov. Commun. Mth.Phys., 7, 289 (1968). Ann.Phys., (N.Y.), 49, 202 (1968).
5. M.K. Volkov. Commun.Math.Phys., 15, 69 (1969).
6. И.С. Градштейн, И.М. Рыжик. "Таблицы интегралов, сумм, рядов и произведений", ГИФМЛ, Москва, 1963 год.

Received by Publishing Department
on June, 22, 1970.