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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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FLAT-SPACE LIMIT OF AN EXACT SOLUTION FOR STATIC GRAVITATIONAL-MASSLESS SCALAR FIELD

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FLAT-SPACE LIMIT OF AN EXACT SOLUTION FOR STATIC GRAVITATIONAL-MASSLESS SCALAR FIELD

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An exact static spherically symmetrical solution of the Einstein equations, when a gravitational and a massless scalar field with a point source are present, was obtained by Fisher in 1948 /1/.

For the Einstein equations $G_\mu^\nu = 8\pi\kappa T_\mu^\nu$ (κ is the gravitation constant) the material tensor of the massless scalar field is

$$ {}_{so}T_\mu^\nu = -\frac{1}{4\pi}\nabla_\mu V \nabla^\nu V + \frac{1}{8\pi}\delta_\mu^\nu \nabla_\sigma V \nabla^\sigma V , $$

and the scalar field equation is of the form

$$ \nabla_\sigma \nabla^\sigma V = 0 . $$

If the metric is

$$ ds^2 = -e^\lambda dr^2 - r^2 [d\theta^2 + \sin^2\theta \, d\phi^2] + e^\nu dt^2, $$

$$ \lambda = \lambda(r), \quad \nu = \nu(r), \quad c = 1 , \qquad (1) $$

and a function $Z(r) \equiv r \exp(\frac{\nu-\lambda}{2})$ is introduced, the Einstein equations yields

$$ r^2 Z^2 Z'' = \kappa G^2 Z', \qquad (2) $$

here G is the integration constant of the scalar field equation, which gives $V' = -G r^{-2} \exp(\frac{\lambda-\nu}{2})$, the prime denotes $\frac{d}{dr}$. The equation (2) can be integrated under the Galilean condition at the space infinity ($e^{\nu} \to 1, e^{\lambda} \to 1$). and the condition for the correspondence with the Newtonian approximation:

$$e^{\nu} = 1 - \frac{2\kappa m}{r} + \dots \quad \text{as} \quad r \to \infty . \tag{3}$$

Let us note that in virtue of (3) the total mass m and (once more) the gravitational constant κ are introduced to the solution. The result of integration is of the form

$$(Z - Z_0)^{1-p} (Z + Z_1)^{1+p} = r^2, \tag{4}$$

here

$$p \equiv \frac{\kappa m}{\sqrt{\kappa^2 m^2 + \kappa G^2}}, \quad Z_{0,1} \equiv \kappa m \left(\frac{1}{p} \mp 1 \right).$$

Now the limit as $\kappa \to 0$ can be reached without any difficulty: $p \to 0$, $Z_{0,1} \to 0$ and consequently, $Z^2 / r^2 \to 1$. With the help of equations $e^{\nu} = \frac{ZZ'}{r}$ and $e^{\lambda} = \frac{rZ'}{Z}$ we deduce $e^{\nu} \to 1$, $e^{\lambda} \to 1$.

That is, the space-time goes over into the flat Galilean one. It is the most natural result when no gravitation is present.

In the paper[2] of Janis, Newman and Winicour a solution is given which coincides with Fisher's one up to coordinate transformation from r to $Z(r)$. [The last is denoted by JNW as $R(r)$]. However they neglect the condition for the correspondence with the Newtonian approximation taking the integration constant $r_0 = 2m$ instead of $2\kappa m$ and obtain $\mu \equiv \frac{1}{p} = \sqrt{1 + \frac{4\kappa G^2}{r_0^2}}$ instead of $\sqrt{1 + \frac{G^2}{\kappa m^2}}$.

So they arrived at the incorrect conclusion that the solution has an essential singularity when $\kappa \to 0$ [x].

[x] This and following remark concern also the recent work of Deser and Higbie[3].

Now let us consider the limit when $G \to 0$ and the scalar field disappears. It would seem that we must obtain the Schwarzschild solution in this limit. However, this transition is really non-unique in the neighbourhood of the origin ($r=0$). Near the origin the solution (4) can be expanded in the form

$$(Z - Z_0) = \left(\frac{2\kappa m}{p}\right)^{\frac{p+1}{p-1}} r^{\frac{2}{1-p}} + \frac{p+1}{p-1}\left(\frac{2\kappa m}{p}\right)^{\frac{p+3}{p-1}} r^{\frac{4}{1-p}} + \dots \qquad (5)$$

It is clear, that there is an essential singularity as $G \to 0$ (i.e. $p \to 1$). But it is also obvious, that the origin is a singular point of the solution and it is like a critical algebraical point. (This singularity was considered some times ago by Markov and the author[4,5]. We have shown that it prevents from the construction of the interior solution at least till the phenomenological pressure is introduced).

At the same time, the point $r=\infty$ is not singular one. Indeed, the Eq. (2) in terms of $y=\frac{Z}{r}$, $x=\frac{1}{r}$ has the form

$$y''_{xx} = (-x\, y'_x + y)\,\frac{\kappa G^2}{y^2}. \qquad (6)$$

If the boundary conditions (3) are fulfilled, that is $y \to 1$, $y'_x \to -2\kappa m$, the point $x=0$ is ordinary and the solution is analytic there (see Cauchy's theorem[6]). Moreover, the limits as $\kappa \to 0$ or as $G \to 0$ can be reached here without difficulty. This is seen also from the expansion

$$\frac{Z}{r} = 1 - \frac{2\kappa m}{r} + \frac{\kappa G^2}{2r^2} + \frac{2}{3}\,\frac{\kappa^2 m G^2}{r^3} + \dots, \quad r \to \infty. \qquad (7)$$

Consequently the exterior part of the Fisher's solution is formally quite correct and unique. Using (7) we obtain

$$e^{\nu} = 1 - \frac{2\kappa m}{r} + \frac{1}{3}\,\frac{\kappa^2 m G^2}{r^3} + \dots, \quad r \to \infty, \qquad (8)$$

from where it follows, that the total mass is equal to m (if Landau formula[7] is used).

In papers[4,5] some possibilities for the physical interpretation of gravitational - scalar field solutions were considered. Introducing, in addition, an electrostatic field and dust-like matter (without pressure) we obtained the complete solution without singularities in r . For instance, in this case it turns out to be impossible to set the total mass zero. Indeed, it is seen from the expansion

$$e^{\nu}=1-\frac{2\kappa m}{r}+\frac{\kappa\epsilon^2}{r^2}+\ldots, \quad r\to\infty, \tag{9}$$

that the Newtonian potential changes his sign when $m\to 0$ (ϵ is the electric charge). It can be shown that the density of the proper mass becomes simultaneously negative in some region of "central body".

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