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Jan T. Łopuszański

PHYSICAL SYMMETRIES IN THE FRAMEWORK OF QUANTUM FIELD THEORY. II

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PHYSICAL SYMMETRIES IN THE FRAMEWORK OF QUANTUM FIELD THEORY. II

Объединенный институт ядерных исследований БИБЛИОТЕКА

* Institute of Theoretical Physics University of Wroclaw, Wroclaw, Poland.

1. Introduction

Our intention is to extend in this note the results previously derived [1] to the most general case of several fields of arbitrary transformation character under the Poincaré group belonging to several nonvanishing masses (see the Appendix). The bound states are also taken into account (see also the Appendix). We consider several locally conserved currents (not necessarily Poincaré invariant, see §3b).

The main result is that the free fields obtained under the symmetry transformation and the original free fields are linked together by a linear, although in general not local, transformation (see §2). The fact that the relations should be linear was conjectured by many physicists although no proof was given so far (see e.g. [2], where some hand waving arguments in favour of such a conjecture are given).

Our statement can be viewed as a proof that non-linear realizations of a symmetry group which have no counterpart in a linear representation cannot be effected in the presence of a mass gap in the theory with interaction. E.g. the three-dimensional non-linear realization of the chiral group $SU(2) \times SU(2)$ cannot be effected since there does not exist an irreducible linear three-dimensional representation (the $(\sigma, \vec{\pi})$ representation in the σ-model is four-dimensional). This is a strong indication that the non-linear realizations are closely related to the appearance of massless particles (possibly to Goldstone's particles) (see e.g.[3]), provided one does not give up locality of the fields and currents or translational invariance of the charges.

We obtained our results using a slightly different technique from that used in previous papers[1]. It is based on using auxiliary fields (used in[4]) like

$$i\,[\,Q\,,\phi\,]\,, \tag{1}$$

where Q denotes the charge and ϕ is the field we started with, instead of fields formerly used by us, viz.

$$V\,\phi\,V^{+} \tag{2}$$

with V unitary. The objects (1) exhibit clear - cut transformation properties under the Poincaré transformation whereas the objects (2) have sometimes an intricate transformation character as e.g.

$$U(\Lambda,a)\,\psi(x,b)\,U(\Lambda,a)^{+} \equiv U(\Lambda,a)\exp\{\,i\,b^{r}\,Q_{r}\,\}\,\phi(x)\exp\{-\,i\,b^{r}\,Q_{r}\,\}U(\Lambda,a)^{+}=$$

$$=\psi\,(\Lambda x+a,\ \Lambda b),$$

where $U(\Lambda,a)$ denotes the unitary Poincaré transformation corresponding to (Λ,a), b_{r} are numbers, Q_{r} - operator charges $r=0,1,2,3$. Moreover, (1) is local with respect to ϕ whilst (2) is not in general; it is even doubtful whether it is always quasilocal or almost local with respect to ϕ.

2. General Considerations

Let us consider a tensorial quantum field of rank $(n+1)$, $n=0,1\ldots$ in the Hilbert space $\mathcal{H}$

$$T_{\lambda_{1}\ldots\lambda_{n+1}}(x)\equiv T_{\lambda}(x) \tag{3a}$$

local, real, transforming under the unitary representation $U(\Lambda,a)$ according to

$$U(\Lambda,a)\,T_{\lambda_{1},\ldots,\lambda_{n+1}}(x)\,U(\Lambda,a)^{+}= \\ =(\Lambda^{-1})^{\mu_{1}}_{\lambda_{1}}\ldots(\Lambda^{-1})^{\mu_{n+1}}_{\lambda_{n+1}}\,T_{\mu_{n_1}\ldots\mu_{n+1}}(\Lambda x+a)\,, \tag{4a}$$

locally conserved

$$\frac{\partial}{\partial x_\mu} T_{\mu,\lambda_1,\ldots,\lambda_n}(x) = 0. \tag{5}$$

Let us assume that the spectrum of the mass operator P^2, where

$$U(1,a) = \exp\{iaP\}$$

consists of two discrete points: $\mu^2 = 0$, corresponding to a unique vacuum, Ω, and $\mu^2 = m^2 \neq 0$, corresponding to the one-particle subspace $\mathcal{H}_1$. This latter assumption is made only for simplicity reasons; we show in the Appendix how to handle the case of several sorts of particles with various, nonvanishing masses. Then

$$Q_{\lambda_1,\ldots,\lambda_n} \equiv \int d^3x\, T_{0,\lambda_1,\ldots,\lambda_n}(x) \tag{6}$$

exists, is translationally invariant and transforms under the Lorentz transformation as a tensor of rank n. We shall call these quantities (6) - charges, although they can but do not need to be considered as generators of a certain group.

Let us assume further that there exists in $\mathcal{H}$ an irreducible set of real fields

$$\phi_\ell^\sigma, \tag{3b}$$

where $\ell = 1,\ldots,k_\sigma$ and σ indicates the transformation character of the field in the Minkowski space, viz.

$$U(\Lambda,a)\,\phi_\ell^\sigma(x)\,U(\Lambda,a)^+ =$$

$$= \sum_\rho S(\Lambda^{-1})^\sigma{}_\rho\, \phi_\ell^\rho(\Lambda x + a), \tag{4b}$$

where $S^\sigma{}_\rho$ is a number matrix.

We demand that these fields are local, local with respect to each other and to T_λ as well as that they have asymptotic limits for $t \to \pm\infty$ belonging to the mass $m^2 \neq 0$ and forming an irreducible set in $\mathcal{H}$. The latter assumption was again made only because of simplicity reasons; in the Appendix we show how to incorporate the bound states into our scheme. We call the asymptotic states

$$\phi^{\sigma}_{\ell,\,ex}(x) \equiv \phi^{\sigma}_{\ell,\,\substack{in\\out}}(x). \tag{7}$$

To avoid trivial cases we assume, without loss of generality, that

$$(\Omega, \phi^{\sigma}_{\ell}(x) E_1\, \phi^{\rho}_{m}(y)\Omega) = i\,\delta_{\ell m}\, \Delta^{(+)\sigma\rho}(x-y\,;\,m^2), \tag{8}$$

where E_1 is the projection operator on $\mathcal{H}_1$, and $\Delta^{(+)\sigma\rho}(x,m^2)$ is the suitable generalization of $\Delta^{(+)}(x;m^2)$ and where $\Delta^{(+)\sigma\rho} = 0$ whenever ϕ^{σ} and ϕ^{ρ} are of different tensorial character. A field $\frac{\partial\phi(x)}{\partial x_{\mu}}$ is not considered as a genuine vector field. Since ϕ^{σ}_{ℓ} and T_{λ} are local and local with respect to each other, they have a common TCP operator, viz.

$$\Theta\,\phi^{\sigma}_{\ell}(x)\Theta^{-1} = \sum_{\rho} \theta^{\sigma}_{\ \rho}\ \phi^{\rho}_{\ell}(-x) \tag{9a}$$

$\theta^{\sigma}_{\ \rho} = \theta^{\sigma}_{\ \rho}$ is a number matrix

$$\Theta\, T_{\lambda_1,\dots,\lambda_{n+1}}\ \Theta^{-1} = (-1)^{n+1}\ T_{\lambda_1,\dots,\ \lambda_{n+1}}(-x). \tag{9b}$$

Thus

$$\Theta\, Q_{\lambda_1,\dots,\lambda_n}\Theta^{-1} = (-1)^{n+1}\ Q_{\lambda_1,\dots,\lambda_n}. \tag{10}$$

Under the assumptions listed above our assertion reads:

i) $$e^{iQ}\,\phi^{\rho}_{\ell,ex}(x)\,e^{-iQ} = \int c^{\rho m}_{\ell,\sigma}(x-y)\,\phi^{\sigma}_{m,\,ex}(y)\,dy, \tag{11a}$$

where

$$Q \equiv \sum_{\alpha} a_{\alpha} Q^{\alpha} = \sum_{r_1,\dots,r_n} a_{r_1}\dots a_{r_n}\ Q^{r_1,\dots,r_n} \tag{11b}$$

a_r -real numbers

ii) $Q^{r_1,\dots,r_n}$ is uniquely defined by (11) as a bilinear form in the incoming (or outgoing) fields,

iii) $[Q, S] = 0$, (12)

where S is the scattering matrix.

To prove our statements i), ii) and iii) let us first of all define the auxiliary real fields

$$i[Q^{a}, \phi^{\sigma}_{m}(x)] \equiv A^{a\sigma}_{m}(x). \tag{13}$$

These fields are local with respect to ϕ^{σ}_{m} since $T^{0,a_1,\dots,a_n}$ is local with respect to them and Q^{a} is translationally invariant. They transform according to the Poincaré representation characterized by the superscripts $a\sigma$. The fields A^{ρ}_{n} acting on the vacuum Ω give a contribution to the one particle states, provided

$Q^{\lambda}\, \mathcal{H}_{1} \equiv 0.$

Then the asymptotic fields

$$i[Q^{a}, \phi^{\sigma}_{m,ex}(x)] = A^{a\sigma}_{m,ex}(x) \tag{14}$$

exist (notice that Q^{a} does not depend on x).

The charges Q^{a} applied to the one particle vectors

$\tilde{\tilde{\phi}}^{\sigma}_{m,ex}(p)\Omega \equiv \Psi^{\sigma}_{m}(p) \qquad p \in V^{-}.$

($\tilde{\tilde{\phi}}$ denotes here the Fourier transform of ϕ)[x] yield

$$\begin{aligned}[Q^{a}, \tilde{\tilde{\phi}}^{\sigma}_{\ell}(p)]\Omega &= Q^{a}\, \Psi^{a}_{\ell}(p) = \\ &= \sum_{n,\rho} k^{a,\sigma,n}_{\ell,\rho}(p)\, \Psi^{\rho}_{n}(p).\end{aligned} \tag{15}$$

We obtained (15) using irreducibility of the fields ϕ on $\mathcal{H}_{1}$ and covariance of Q^{a} under the Poincaré transformation. Q^{a} annihilates the vacuum since the additive constant numbers Q^{a} vanish for $n = 0,2,4$ because of the TCP invariance, for other n because of the tensorial character of Q^{a}. Because of TCP covariance

[x] Due to the condition (8) and the stability of the one-particle states the latter are orthogonal to each other for different lower indices and for upper indices characterizing fields of different transformation character.

$$(-1)^{n+1} \sum_{\rho} \bar{k}_{\ell,\rho}^{a;\dot{\sigma},n}(p)\,\theta^{\rho}_{\lambda} = \sum_{\rho} \theta^{\sigma}_{\rho}\, k_{\ell,\lambda}^{a;\rho,n}(p) \tag{16a}$$

and because of the hermiticity of Q^{a}

$$\sum_{\lambda} k_{\ell,\lambda}^{a,\sigma,n} F^{\omega\lambda}(p) = \sum_{\lambda} k_{n,\lambda}^{a,\omega,\ell} F^{\lambda\sigma}(p), \tag{16b}$$

with

$$2\omega_{p}\,\delta(\vec{p}-\vec{q})F^{\omega\lambda}(p) \equiv (\Psi^{\omega}_{n}(p), \Psi^{\lambda}_{n}(q)). \tag{16c}$$

On one hand the generalized result of Borchers /5/ asserts that

$$[\tilde{\tilde{\phi}}^{\rho}_{,ex}(p), \tilde{\tilde{A}}^{a\sigma}_{\ell,ex}(q)] = \text{number}. \tag{17}$$

Taking the vacuum expectation value of (17) for $p \in V^{+}$ and $g \in V^{-}$ and making use of (15) and (16c) we obtain

$$\begin{aligned}[\tilde{\tilde{\phi}}^{\rho}_{n,ex}(p), A^{a\sigma}_{\ell,ex}(q)] &= \\ &= i \sum_{\lambda} k_{\ell,\lambda}^{a;\sigma,n}(p) F^{\rho\lambda}\, 2\omega_{p}\,\delta(\vec{p}-\vec{q}).\end{aligned} \tag{18}$$

On the other hand the same result as in (18) follows by the ansatz

$$\begin{aligned}\tilde{\tilde{A}}^{a\sigma}_{\ell,ex}(q) &\equiv i[Q^{a}, \tilde{\tilde{\phi}}^{\sigma}_{\ell,ex}(q)] = \\ &= i \sum_{n,\rho} k_{\ell,\rho}^{a,\sigma,n}(q)\, \tilde{\tilde{\phi}}^{\rho}_{n}(q)\end{aligned} \tag{19}$$

with q still in V^{-}. Because of the irreducibility of the fields ϕ_{a} as well as because of the translational invariance of Q relation (19) is the right choice. The relation (19) can now be extended to $q \in V^{+}$ and $p \in V^{-}$ by using the hermiticity of the fielfs ϕ and A. This implies automatically

$$k^{\alpha,\sigma\ ,n}_{\ell,\ \rho}(p) = -\bar{k}^{\alpha,\sigma,n}_{\ell,\ \rho}(-p) \tag{16d}$$

Q^{α} is uniquely defined by (19) in virtue of the irreducibility of the fields ϕ_{ex} as well as of Q^{α} annihilating the vacuum. It follows immediately from (19) that Q^{α} is a bilinear form in the incoming (or outgoing) fields. Q^{α} commutes with the scattering matrix since (19) holds for incoming and outgoing fields as well.

Since we know how Q acts on ϕ_{ex} we conclude that

$$e^{iQ}\tilde{\tilde{\phi}}^{\rho}_{\ell,\,ex}(q)\,e^{-iQ} = \Sigma\,\tilde{\tilde{c}}^{\rho,\,n}_{\ell,\ \lambda}(q)\phi^{\lambda}_{n,\,ex}(q), \tag{20}$$

where Q is given by (11b), and

$$\tilde{\tilde{c}}^{\rho,\,n}_{\ell,\ \lambda} \equiv (e^{ik})^{\rho,\,n}_{\ell,\ \lambda} \qquad \text{(20a)}^{x/}$$

$$k \equiv \sum_{\alpha} a_{\alpha} k^{\alpha} = \sum_{r_1 \dots r_n} a_{r_1} \dots a_{r_n} k^{r_1 \dots r_n},$$

From (20) follows (11) by taking the inverse Fourier transform. Since ϕ are real fields, c is real too. The r.h.s. of (11) has to satisfy the K.G. Equation with the mass m ; therefore

$$\int_S \left(\tilde{\tilde{c}}(x-y)\frac{\partial\phi(y)}{\partial r_y} - \frac{\tilde{\tilde{c}}(x-y)}{\partial r_y}\phi(y)\right)dy \to 0$$

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x/ To get the result (20) we have only to iterate the formula (19); e.g. with Q given by (11a) we have

$$[Q,[Q,\tilde{\tilde{\phi}}^{\sigma}_{\ell,\,ex}(q)]] = \sum_{n,\rho} k^{\sigma,\,n}_{\ell,\ \rho}(q)[Q,\tilde{\tilde{\phi}}^{\rho}_{,n,ex}(q)] =$$

$$= \sum_{n,\rho}\sum_{s,\tau} k^{\sigma,\,n}_{\ell,\ \rho}\, k^{\rho,\,s}_{n,\ \tau}\, \tilde{\tilde{\phi}}^{\tau}_{s,\,ex}(q), \text{etc.}$$

wherever the 3-dimensional boundary S of the 4-dimensional volume tends to infinity. Presumably the l.h.s. of (11) is almost local with respect to ϕ ; then $c(x)$ has to fall off very rapidly with $\vec{x}\to\infty$.

3. Example of Vector and Tensor Charges

a). Application of the Assertion

Let us confine ourselves in the sequel for the sake of simplicity to the case of ℓ real fields ϕ_j, $j=1,..,\ell$. In this particular case $v^{\sigma}{}_{\rho}$ in (9a) is just a unit matrix.

For $n=1$ we have

$$[Q_r\, , \tilde{\tilde{\phi}}_{s,ex}(q)] = \sum_j k^j_{r,s}(q)\tilde{\tilde{\phi}}_j(q) \qquad s=1,\dots,\ell, \tag{21}$$

where

$$k^j_{r,s}(q) = q_r a^j_s \tag{21a}$$

and

$$a_s{}^j = \overline{a_s{}^j} = a_j{}^s \tag{21b}$$

(16d) is satisfied automatically. We may diagonalize the matrix $a_j{}^s$ by a nonsingular matrix $v_j{}^s$. For the fields

$$\tilde{\tilde{\Psi}}_{s,ex}(q) = \sum_{g=1}^{\ell} v_s{}^j \tilde{\tilde{\phi}}_{j,ex}(q)$$

(21) takes the form

$$[Q_r\, , \tilde{\tilde{\Psi}}_{s,ex}(q)] = \lambda_s q_r \tilde{\tilde{\Psi}}_{s,ex}(q), \tag{22}$$

where λ_s are the (real) eigenvalues of $a_s{}^j$. The fields $\Psi_{s,ex}$, $s=1,2,..\ell$, form also an irreducible set. Should the theory of the fields Ψ be non-trivial we expect to have nonvanishing matrix element between the outgoing and incoming states describing each different set of particles, as pointed out in [4], e.g.

$$(\tilde{\tilde{\Psi}}^{+}_{j_1,out}(p)\dots\tilde{\tilde{\Psi}}^{+}_{j_m,out}(p\ )\Omega,\tilde{\tilde{\Psi}}^{+}_{k_1,in}(q_1)\dots\tilde{\tilde{\Psi}}^{+}_{k_n,in}(q_n)\Omega)\neq 0 \qquad p,q\in V^{+} \tag{23a}$$

for some m and n large than one and some sets of $\{j\}$ and $\{k\}$. According to (22) and using the hermiticity of Q_r we find that

$$(\prod_{r=1}^{m}\tilde{\tilde{\Psi}}^{+}_{j_r,out}(p_r)\Omega,Q_r\prod_{s=1}^{n}\tilde{\tilde{\Psi}}^{+}_{k_s,in}(q_s)\Omega)= \tag{23b}$$

$$=(\sum_{r=1}^{m}\lambda_{j_r}p_r)(\prod_{r=1}^{m}\tilde{\tilde{\Psi}}^{+}_{j_r,out}(p_r)\Omega,\prod_{s=1}^{n}\tilde{\tilde{\Psi}}^{+}_{k_s,in}(q_s)\Omega)= \tag{23c}$$

$$=(\sum_{s=1}^{n}\lambda_{k_s}q_s)(\prod_{s=1}^{m}\tilde{\tilde{\Psi}}^{+}_{j_r,out}(p_r)\Omega,\prod_{s=1}^{n}\tilde{\tilde{\Psi}}^{+}_{k_s,in}(q_s)\Omega). \tag{23d}$$

Of course,

$$\sum_{r=1}^{m}p_r=\sum_{s=1}^{n}q_s. \tag{23e}$$

Since the equalities (23c,d) and 23e) hold for a non-enumerable set of q's and p's and (23a) is assumed to be satisfied we conclude that λ_s does not depend on the index s. Thus, writing $\lambda_s=\lambda$, (21a) reduces to

$$k_{r,s}^{\ j}(q)=\lambda q_r\delta_s^{\ j} \tag{24a}$$

so that ϕ_{ex} as well as Ψ_{ex} satisfy the same relation (22) with λ_s replaced by λ. Moreover,

$$Q_r=\lambda P_r. \tag{24b}$$

In words: the only admissible tensor currents of rank two are those giving rise - up to a numerical factor - to the generators of the translation group. This confirms the result stated in/4/ and suggested by Sugawara/6/.

For $n=2$ we have

$$[Q_{\mu\nu}, \tilde{\tilde{\phi}}_{s,ex}(q)] = \sum_{j=1}^{\ell} k_{\mu\nu,s}^{\;j}(q)\, \tilde{\tilde{\phi}}_{j,ex}(q), \tag{25a}$$

where

$$k_{\mu\nu,s}^{\;j}(q) \equiv a_s^{\;j} q_\mu q_\nu + b_s^{\;j} g_{\mu\nu} \tag{25b}$$

and $a_s^{\;j}$ and $a_s^{\;j}$ are both imaginary and antisymmetrical (hermitean) matrices. (16d) is again automatically satisfied. Because of the commutation relations for the free fields and the irreducibility of these fields

$$Q_{\mu\nu} = -\sum_{s,j} \int k_{\mu\nu,s}^{\;j}(q)\, \tilde{\tilde{\phi}}_s^{+}(q)\, \tilde{\tilde{\phi}}_j(q)\, \delta(q^2 - m^2)\, \theta(q)\, dq \; .$$

If $a_s^{\;j}$ and $b_s^{\;j}$ can be diagonalized simultaneously the algebra of $Q_{\mu\nu}$ will be again abelian. In this case a similar consideration to that presented for the case $n=1$ (see eq. (23)) brings us to the conclusion that

$$k_{\mu\nu,s}^{\;j}(q) = 0 \tag{26a}$$

or

$$Q_{\mu\nu} = 0. \tag{26b}$$

This result (26) can be generalized and we are entitled to claim that the abelian tensor charges of rank not less than two vanish. Thus only the case of abelian scalar charges offers some interest (gauge transformations, superselection rules).

If, however, the matrices $a_s^{\;j}$ and $b_s^{\;j}$ do not commute with each other and in addition

$$k_{\mu\nu} k_{\kappa\lambda} - k_{\kappa\lambda} k_{\mu\nu} = t_{\mu\nu,\kappa\lambda}^{\rho\sigma} \; k_{\rho\sigma}$$

with $\quad t_{\mu\nu,\kappa\lambda}^{\rho\sigma} = - t_{\kappa\lambda,\mu\nu}^{\rho\sigma} = - \bar{t}_{\mu\nu,\kappa\lambda}^{\rho\sigma}$

then

$$[Q_{\mu\nu}, Q_{\kappa\lambda}] = -A^{\rho\sigma}_{\mu\nu,\kappa\lambda} \; Q_{\rho\sigma} .$$

In this case as well as in other cases of non-abelian tensorial charges of rank not less than two we are not able to predict anything definite about the structure of the charges.

b) The Case of Currents of No Pure Tensor Character

For the sake of completeness let us look to the case of locally conserved currents which do no longer transform according to (4a) but, nevertheless, the integration performed in (6) results in charges which transform already like (pseudo) tensors. An example of paramount importance are the Lorentz group generators

$$M_{\mu\nu} = \int d^3x \, (x_\mu P_{\nu 0}(x) - x_\nu P_{\mu 0}(x)) ;$$

$P_{\mu\nu}(x)$ - energy-momentum tensor.

More generally, to every locally conserved tensor current $T_{\mu\nu} = \pm T_{\nu\mu}$ corresponds another locally conserved current

$$T_{\mu,\nu\lambda} = x_\nu T_{\lambda\mu} \mp x_\lambda T_{\nu\mu} \tag{27}$$

which, however, does not display pure tensor properties (see /7/). In particular, it is not translationally invariant. Under the Lorentz transformation

$$\hat{Q}_{\mu\nu} \equiv \int d^3x \, (x_\mu T_{\nu 0} \mp x_\nu T_{\mu 0}) \tag{28}$$

transform as a tensor of the 2nd rank. However,

$$i\,[P_\mu, \hat{Q}_{\nu\lambda}] = -g_{\mu\nu} Q_\lambda \pm g_{\mu\lambda} Q_\nu . \tag{29}$$

Thus $Q_{\mu\nu}$ do not commute with the translations, although they are time independent (due to local conservation). One can easily check that

$$[P_\kappa, [P_\lambda, \hat{Q}_{\mu\nu}]] = 0. \tag{30}$$

In virtue of (30) $\hat{Q}_{\mu\nu}$ transform one particle states into one particle states of the same mass m. Since we can not exploit the commutability of $Q_{\mu\nu}$ with P_r, our final result reads

$$[\hat{Q}_{\mu\nu}, \tilde{\tilde{\phi}}_{j,ex}(p)] = i \sum_{s=1}^{\ell} \int \tilde{\tilde{K}}_{js,\mu\nu}(p,q) \tilde{\tilde{\phi}}_{s,ex}(q) \delta(q^2 - m^2) \theta(q) dq, \tag{31a}$$

where $\tilde{\tilde{K}}$ is a real function and

$$\tilde{\tilde{K}}_{mn,\mu\nu}(p,q) = -\tilde{\tilde{K}}_{nm,\mu\nu}(q,p). \tag{31b}$$

Formula (31) shows that the relation remains still linear[x].

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Appendix

Let us concentrate - for simplicity reasons - again on the case of n scalar real fields $\phi_j(x)$ $j = 1, \ldots, n$. These fields are supposed to conform with all the requirments imposed upon them in §2 except that $\phi_{j,ex}$ belonging to mass $m \neq 0$ do not form an irreducible set whereas ϕ_j themselves do.

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[x] When this note was accomplished we got a preprint/8/ where it is shown under very mild assumptions that $\hat{Q}_{\mu\nu} = cM_{\mu\nu}$, c - a number. This makes the above example trivial. Nevertheless, the method indicated by us can easily be extended to cases when the tensor current T is of higher rank than two.

There are still other free fields, let us call them $\beta_{t,ex}$ $t=1,..,k$ belonging to a mass M different[x] from m and from zero which give rise to one particle states and span the Hilbert subspace $\mathcal{H}_M$. The fields $\beta_{t,ex}$ are asymptotic for the fields β_t. Then

$$\sum_{t=1}^{k} b_t E_M \beta_t(f)\Omega = 0 \qquad \text{if} \quad b_t = 0, \tag{A.1}$$

where E_m and E_M are the projection operators on $\mathcal{H}_m$ and $\mathcal{H}_M$ respectively. Both sets, $\beta_{t,ex}$ and β_t, can be constructed out of the fields ϕ_j (in virtue of the irreducibility of the latter). We assume that β_t are local with respect to ϕ_j

$$[\beta_t(x), \phi_j(y)] = 0 \qquad (x-y)^2 < 0$$
$$t = 1,..,k\,;\; j = 1, .., n. \tag{A.2a}$$

Hence follows

$$[\beta_t(x), T_\lambda(y)] = 0 \quad (x-y)^2 < 0 \quad \lambda = 0,1,2,3. \tag{A.2b}$$

as well as that β_t are local themselves too. We assume further that

$$(\Omega, \beta_t(x) E_M \beta_s(y)\Omega) = i\,\delta_{ts}\,\Delta^{(+)}(x-y\,; M^2) \qquad t,s = 1,..,k \tag{A.3}$$

as well as

$$E_M \phi_j(f)\Omega = 0 \tag{A.4a}$$

$$E_m \beta_t(f)\Omega = 0. \tag{A.4b}$$

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[x] The difference in the masses m and M is not essential but we keep it for notation convenience.

The last assumption is the same as to say that ϕ_j and β_t are irreducibly represented in $\mathcal{H}_m$ and $\mathcal{H}_M$ resp. In addition β_t should transform under the same Lorentz representation as ϕ_j and have the same TCP operator as the latter.

Let us consider the simplest case of Q being a (pseudo) scalar. For $\Phi_j(f) \equiv \phi_{j,ex}(f)\Omega$ and $\beta_s(f) \equiv \beta_{s,ex}(f)\Omega$ x/ we have

$$iQ\Phi_j(f) = \sum_{k=1}^{n} a_{jk}\Phi_k(f) + \xi_j(f, \beta_{ex}, m^2) \tag{A.5a}$$

$$iQ\beta_t(g) = \sum_{s=1}^{k} \gamma_{ts}\beta_s(f) + \eta_t(f, \phi_{ex}, M^2), \tag{A.5b}$$

where the matrices a and γ are real and skewsymmetric of dimensions n and k resp. and ξ_j, η_t depend only on β'_{ex} and ϕ_{ex} resp. and are orthogonal to Φ_k as well as to β_s.

Let us apply the Borchers's theorem (see also/7/ to the auxiliary fields

$$i[Q, \phi_j(x)] \equiv A_j(x) \tag{A.6a}$$

$$i[Q, \beta_t(x)] \equiv B_t(x). \tag{A.6b}$$

Both sets are local with respect to ϕ and β and contribute to the one particle states. We get

$$\left.\begin{aligned}
&[\phi_{j,ex}(x), A_{k,ex}(y)] = &\text{(A.7a)}\\
&[\phi_{j,ex}(x), B_{s,ex}(y)] = &\text{(A.7b)}\\
&[\beta_{t,ex}(x), A_{k,ex}(y)] = &\text{(A.7c)}\\
&[\beta_{t,ex}(x), B_{s,ex}(y)] = &\text{(A.7d)}
\end{aligned}\right\}\ \text{a number}$$

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x/ Notice that $(\Phi_j(f), \Phi_k(g)) = (\Phi_j(f), \beta_t(g)) = (\beta_s(f), \beta_t(g)) = 0$ for $j \neq k$ and $t \neq s$.

In order to verify the formulae

$$i\,[\,Q\,,\phi_{j,ex}(x)] = \sum_{k=1}^{n} a_{jk}\ \phi_{j,ex}(x) \tag{A.8a}$$

$$i\,[\,Q\ \ ,\beta_{t,ex}(x)] = \sum_{s=1}^{k} \gamma_{ts}\ \beta_{s,ex}(x) \tag{A.8b}$$

one has to confront the vacuum expectation values obtained on one hand by using (A.7), (A.6) and the orthogonality relations for the one particle states, on the other hand directly by substituting (A.8) into (A.7) and using the canonical commutation relations. It turns out that the states ξ and η are equal to zero.

A similar scheme may be adopted in case of two sorts of particles of mass m and M (no bound states). In this case (A.1) will be replaced by

$$\sum_{\ell=1}^{m} a_\ell E_m \phi_\ell(f)\Omega = 0 \qquad \text{iff} \quad a_\ell = 0\,, \tag{A.9a}$$

$$\sum_{\ell=1}^{n} b_\ell E_M \phi_\ell(f)\Omega = 0 \qquad \text{iff} \quad b_\ell = 0\,. \tag{A.9b}$$

The fields $\phi_{\ell,ex}(x;m^2)$ and $\phi_{\ell,ex}(x;M^2)$ form an irreducible set. Instead of (A.3) we have

$$(\Omega,\phi_k(x)E_m\phi_\ell(y)\Omega) = i\,\sigma^{m}_{k\ell}\ \Delta^{(+)}(x-y\,;m^2) \tag{A.10a}$$

$$(\Omega,\phi_k(x)E_M\phi_\ell(y)\Omega) = i\,\sigma^{M}_{k\ell}\ \Delta^{(+)}(x-y\ \ ;M^2), \tag{A.10b}$$

where $\sigma^{m,M}_{kk} \geq 0$ (positive definite metric) and $\sigma^{m,M}_{k\ell} = \overline{\sigma^{m,M}_{k\ell}}$ (TCP covariance). It is, in general, not possible to perform simultaneously the "diagonalization" of the one particle states in both spaces $\mathcal{H}_n$ and $\mathcal{H}_M$. This manifests itself in the non-standard commutation relations of the fields $\phi_{\ell,ex}(x;m^2,M^2)$. Nevertheless we may perform the diagonalization of the one particle states in every subspace separately. Then the corresponding free fields $\hat{\phi}_{\ell,ex}$ being linear combinations of $\phi_{\ell,ex}(m^2,M^2)$, will satisfy the normal canonical commutation relations.

The field (A.6a) has also two limits depending on the test function used, namely $A_{j,ex}(x;m^2)$ and $A_{j,ex}(x;M^2)$. The application of the Borchers's theorem leads us to

$$i[Q,\hat{\phi}_{j,ex}(x;m^2)] = \sum_{k=1}^{m} a^{m}_{jk}\,\hat{\phi}_{k,ex}(x;m^2) \tag{A.11a}$$

$$i[Q,\hat{\phi}_{j,ex}(x;M^2)] = \sum_{k=1}^{n} a^{M}_{jk}\,\hat{\phi}_{k,ex}(x;M^2) \tag{A.11b}$$

where $a^{m,M}_{jk}$ are real and skewsymmetric matrices.

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