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As is known, the matter density in a collapsing electrically neutral system increases indefinitely tending to an infinite value[1]. Thus unusual state of matter with an infinite density and further destiny of the collapse is often the subject of discussions, more exactly, the subject of questions.

The usual answer to the question which makes the interest for this problem not so keen is related to the arguments according to which the state of the matter with the density exceeding the value[1]

$$\rho_q \approx 10^{93} \ \mathrm{g/cm^3} \qquad (1)$$

is not described by modern theory.

Thus, it is implied that a further theory of gravitation including also the description of the quantum properties of the gravitational field will naturally answer this question, too.

In 1960 Graves and Brill[2] generalized the well-known Kruscal's transformation to the case of the Nordstrőm-Reissner metric. In these coordinates the pseudo-singularities of the well-known external Nordströ́m-Reissner solution disappear and thus, a complete description of the Nordströ́m-Reissner space is given.

As in the corresponding Kruscal's case the given metric turns out to be non-stationary. In the present case it is of oscillatory character.

Analysing the results of paper /2/ I.D.Novikov /3/ has paid attention to the fact that in the case of the collapse of an electrically charged system the gravitational contraction gives place to an extension x/ for the finite value of the density matter.

Usually one considers the collapse of electrically neutral matter. In the present note we would like to draw attention to the fact that in this case the electrically neutral vector meson fields (μ: ρ, ϕ, ω) stop also collapsing of stars at densities much smaller than $\rho_q \approx 10^{93}$ g/cm^3 and the gravitational contraction changes into the extension.

At first sight it seems that the forces, being short-range ($\phi \approx g_\mu e^{-\frac{m_\mu c}{\hbar} r}$) cannot be of much importance in cosmic systems.

Indeed, the role of the short-range forces in the development of the collapse was repeatedly discussed. But usually one considered the case when the dimensions of a collapsing system are much larger than the region of action of nuclear forces ($\frac{\hbar}{m_\mu c}$).

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x/ The system is extended beyond the gravitational radius in the region of space which lies in an absolute future with respect to the space where the system was collapsing.

In this case it is possible to introduce the notion of pressure since the nuclear energy under consideration enters additively the volumes of the system.

A general thermodynamic consideration shows that at this stage of development of the collapse ($R \gg \hbar/m_\mu c$) nuclear forces do not stop collapsing of the system.

However, for $R \leq \hbar / m_\mu c$ the energy of the particles of the system is no longer additive i.e. the phenomenon goes beyond the framework of the usual thermodynamic treatment and further collapsing process should be considered from the dynamic point of view.

This stage of development of the collapse is of interest since when the system acquires such small dimensions the matter density remains smaller than the critical one. Indeed, the collapsing mass

$$M \approx 2 M_\odot \approx 4 . 10^{33} \text{ g} ,$$

which is concentrated in the volume $\frac{4}{3}\pi\left(\frac{\hbar}{m_\mu c}\right)^3 \approx 10^{-40} \text{ cm}^3$ possesses the density

$$\rho = \frac{6 M_\odot}{4\pi} \left(\frac{m_\mu c}{\hbar}\right)^3 \approx 10^{73} \text{ g/cm}^3 , \qquad (2)$$

which is by 20 orders of magnitude smaller than $\rho_q \approx 10^{93}$ g/cm^3.

When the system acquires nuclear sizes the qualitative picture of stopping of the collapse looks as follows.

When the dimensions of the system decrease the gravitational mass defect grows and thereby decreases the total mass but at the same time the energy of the vector meson field increases which is more and more localized in the space outside matter: when $R \to \frac{\hbar}{m_\mu c}$

this energy becomes so large that with further decreases of the dimensions of the system it would exceed the total initial mass of the collapsing system which is impossible since the initial energy of the system must remain unchanged.

Indeed, if g_ρ is the ρ -meson charge of a neutron and if the system consists of N neutrons then the energy of the ρ -meson field for $R \lessapprox \hbar / m_\mu c$ is

$$E_\rho \approx \frac{g_\rho^2 N^2}{r} . \tag{3}$$

If a collapsing star has the mass $N m_n$ i.e. its total energy is

$$E_s \approx N m_n c^2 , \tag{4}$$

then for $r \approx \hbar / m_\mu c$

$$E_\rho' > E_s ,$$

or

$$g_\rho^2 \frac{N^2}{\hbar} m_\rho c >> N m_n c^2 , \tag{5}$$

or putting $m_\rho \approx m_n$

$$\frac{g^2}{\hbar c} N >> 1 , \quad \text{since} \quad \frac{g^2}{\hbar c} \approx 1 . \tag{6}$$

These rather non-rigorous estimates illustrate the assertion according to which the collapsing system consisting of matter cannot reduce up to the following dimensions:

$$R < \frac{\hbar}{m_\mu c} .$$

A more rigorous argument[x] consists in that for the dimensions of the collapsing system $R < \hbar / m_{\mu} c$ the forces are no longer short-range for the matter in question. All the processes may be argued in the same manner as in the case of the collapse stopping by a distribution of the electric charge of the system /3/.

More detailed, but now somewhat underestimated, values which are considered in what follows lead also to stopping of the collapse for these small dimensions of the system.

Indeed, let the collapse of a certain critical mass (let $M = 2 M_{\odot}$) proceed under the action of only gravitational forces But when the collapsing system reaches dimensions of the order $r_0 = \hbar / m_{\mu} c$ repulsive forces (we assume for simplicity only ρ -meson forces) are switched on which at these distances $r < r_0$ are approximated by the potential

$$\phi \approx \frac{\epsilon}{r} , \qquad \text{with } \epsilon = g_{\rho} (N_n - N_{\tilde{n}}) ,$$

here g_{ρ} is the ρ - meson charge of the neutron, N_n and $N_{\tilde{n}}$ are the numbers of neutrons and antineutrons of the system, respectively.

Let us look for the motion of a real particle, e.g. a neutron, located on the boundary matter-vacuum.

Let the radial motion of this neutron begin at the distance R then its velocity at the moment when it reaches the radius r_0

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[x] The non-rigorousness of the consideration performed consists in that it should be made consistently, in the general relativity formalism. In contrast to classical physics e.g. the self-energy of the source of ρ field in the general relativity theory is not divergent. This gives overestimated values of the meson field effect, firther underestimated values are given.

is given by the first integral of motion

$$\left(\frac{dr}{d\tau} \right)^2 = \frac{2\kappa M}{r} \left(\frac{1}{r_0} - \frac{1}{R} \right) , \qquad (7)$$

where κ is the gravitational constant, M is the Schwarzschild mass of the collapsing system. At this moment forces with the potential

$$\phi = \frac{g_\rho (N_n - N_{\tilde{n}})}{r}$$

are switched on.

Further motion of the neutron proceeds in the Nordström-Reissner field.

Now for the first integral we can rewrite the expression given by Graves and Brill /2/ with replacement of the electric charge of the system by the ρ -meson field charge

$$k^2 - 1 = \left(\frac{dr}{d\tau}\right)^2 + 2\frac{-\frac{M\kappa}{c^2} + \frac{\epsilon\kappa^2}{c^2}\beta k}{r} + (1-\beta^2)\frac{\epsilon^2 \kappa}{c^4 r^2} \qquad (8)$$

Here k is the constant of integration, M is the external Schwarzschild mass of a collapsing body, ϵ its total ρ -meson charge

$$\epsilon = g_\rho (N_n - N_{\tilde{n}}) ,$$

$$\beta = \frac{g_\rho^2}{\epsilon m_n^2} \approx \frac{\hbar c}{\epsilon m_n^2} , \qquad (9)$$

m_n is the neutron mass (test particle), τ is the proper time.

Inserting in (8) the value (7)

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{2\kappa M}{c^2}\left(\frac{1}{r_0} - \frac{1}{R}\right),$$

we find

$$k = \frac{\epsilon\,\kappa^{1/2}}{c^2\,r_0}\beta + \sqrt{1 + \frac{\epsilon^2\,\kappa^2}{c^4\,r_0^2} - \frac{2M\kappa}{c^2\,R}}\,. \tag{10}$$

For $\beta \gg 1$ and $R \to \infty$ (11)

$$k \approx \frac{\epsilon\,\kappa^{1/2}}{c^2\,r_0}\,\beta \gg 1\,. \tag{12}$$

At the moment of stopping of a test particle under the action of ρ -forces the expression $dr/d\tau$ should vanish

$$\frac{dr}{d\tau} = 0\,. \tag{13}$$

Taking into account (11), (12), (13) at this moment the relations (8) can be rewritten in the form

$$r^2 k^2 - 2r\left(-\frac{M\kappa}{c^2} + \frac{\epsilon\sqrt{\kappa}}{c^2}\beta k\right) + \frac{\beta^2\,\epsilon^2\,\kappa^2}{c^4} = 0\,. \tag{14}$$

If we assume

$$\frac{M\kappa}{c^2} \ll \frac{\epsilon\sqrt{\kappa}}{c^2}\beta k\,, \tag{15}$$

this may be also fulfilled under the condition[x]

$$M \gg \frac{\epsilon}{\sqrt{\kappa}} \qquad \text{for} \quad \beta k \gg 1 \ , \tag{16}$$

then

$$r^2 k^2 - \frac{2 r k \epsilon \sqrt{\kappa}\, \beta}{c^2} + \frac{\beta^2 \epsilon^2 \kappa}{c^4} = 0 \ , \tag{17}$$

or

$$\left(r k - \beta \epsilon \frac{\sqrt{\kappa}}{c^2} \right)^2 = 0 \ .$$

Thus, at the moment of stopping of the test particle

$$r_s = \frac{\beta \epsilon \sqrt{\kappa}}{c^2 k} \ , \tag{18}$$

or inserting for k the expression (12) we get

$$r_s \approx r_0 \ . \tag{19}$$

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[x] i.e. $\epsilon \frac{\sqrt{\kappa}}{c^2} < \frac{M\kappa}{c^2} \ll \frac{\epsilon\sqrt{\kappa}}{c^2} \beta k$, or
$g_\rho (N_n - N_{\tilde{n}}) \sqrt{\kappa} < m_n k (N_n + N_{\tilde{n}}) < g_\rho (N_n - N_{\tilde{n}}) \beta k \sqrt{\kappa}$,
this condition is not due to physical restrictions, it enables us to use the ready equation (8) written for the case of small charges $M > \epsilon/\sqrt{\kappa}$. This fact makes our values underestimated, since the collapse stopes for large $N_{\tilde{n}}$ as well.

Thus, under the action of short-range meson forces the gravitational collapse of the critical mass stops when the collapsing system reaches the dimensions $\hbar / m_\rho c$.

This result is obtained in a wide range of the values $\epsilon = g_\rho (N_n - N_{\tilde{n}})$. The result is all the more true for the real values of $N_n - N_{\tilde{n}}$ in celestial bodies.

It should be stressed that the densities $\rho_q \approx 10^{93}$ g/cm^3 for the underestimated values would be reached for collapsing masses $m \approx \rho_q \left(\frac{\hbar}{m_\rho c} \right)^3 \approx 10^{20} M_\odot \approx 10^{54}$ g i.e. of the order of the Universe mass.

These rough estimates make it probable to state that meson forces can stop not only the collapse of celestial bodies but also the collapse of the Universe and the limiting density of the latter may not reach the quantum densities ρ_q.

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