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DEEP INELASTIC LEPTON-HADRON SCATTERING AND PAIR PRODUCTION IN STRONG INTERACTIONS

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DEEP INELASTIC LEPTON-HADRON SCATTERING AND PAIR PRODUCTION IN STRONG INTERACTIONS

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Глубоко неупругое лептон- адронное рассеяние
и образование пар в сильных взаимодействиях

Мы рассматриваем глубоко неупругое рассеяние лептона (ℓ -электрон или мюон) на адроне a (a -протон или нейтрон), когда в конечном состоянии детектируется рассеянный лептон и некоторый адрон

$$\ell + a \to \ell + a' + \text{"адроны"}$$

и "адроны" указывает на то, что подразумевается суммирование по всем другим возможным адронам. Пять структурных функций этого процесса связываются с коммутаторами электромагнитного тока. При помощи кроссинга мы находим связь с процессом образования лептонной пары в сильных взаимодействиях

$$\bar{a}' + a \to \ell + \text{"адроны"}$$

Предложена проверка моделей для составляющих электромагнитного тока при помощи идеализированной модели кварков.

Сообщения Объединенного института ядерных исследований

Дубна, 1969

Kögerler R.F., Muradyan R.M. E2-4791

Deep Inelastic Lepton-Hadron Scattering and Pair
Production in Strong Interactions

We consider the deep inelastic lepton (ℓ -electron or muon) scattering on a hadron a (a -proton or neutron) when in final state the outgoing lepton and some hadron a' are detected

$$\ell + a \to \ell + a' + \text{"anything"}$$

and "anything" indicates the sum over all other possible hadrons. The five structure functions for this process are connected with the electromagnetic current commutators. By means of crossing we find the connection with the process of lepton pair production in strong interactions

$$\bar{a}' + a \to \ell + \bar{\ell} + \text{"anything"}$$

A test of models for the constituents of the electromagnetic current is made using a very simplified quark model.

Communications of the Joint Institute for Nuclear Research.

Dubna, 1969

1. Introduction

It has become clear that the study of deep inelastic processes by using electromagnetic or weak currents as a probe can give valuable information about the structure of hadrons at high energies and large q^2 (timelike or spacelike).

Here we consider the process of deep inelastic lepton-hadron scattering with one hadron in the final state singled out

$$\ell + a \to \ell + a' + \text{"anything"} , \tag{1.1}$$

where ℓ denotes the lepton (electron or muon); a and a' stand for hadrons; "anything" indicates the sum over all other possible hadronic states which are permitted by the conservation laws. Usually the target hadron a is a proton or a neutron; the outgoing hadron a' may be a proton, antiproton, pion, etc.

By means of the crossing properties of field theory this reaction is closely connected with the process of lepton pair production in deep inelastic hadron-hadron collisions

$$\bar{a}' + a \to \ell + \bar{\ell} + \text{"anything"} . \tag{1.2}$$

It has been pointed out that both from the theoretical /1/ and experimental /2/ standpoint the investigation of this process will be of particular interest. In analogy with this process the high energy limit of the five structure functions which describe the hadronic structure of process (1.1) is found and the connection with equal time electromagnetic current commutators is obtained. Finally, it is pointed out how the different models for the constituents of the electromagnetic current can be distinguished by means of these ETCR.

It is clear that the process (1.1) is a more general case of the usual electroproduction process, when in the final state only the scattered lepton is detected

$$\ell + a \to \ell + \text{"anything"}, \tag{1.3}$$

where the target particle a is a proton or a neutron. This process has been treated in great detail in the past two years. A survey of the present status of these investigations can be found, for example, in Bjorken's review /3/. Note that the connection of this process with electron-positron annihilation has been studied recently by Drell, Levy and Yan /4/ in a special field-theoretical model (see also /7/).

In section II the kinematic analysis is presented. In section III the connection with lepton pair production is considered. In section IV the relation between the high energy limit of the structure functions and current commutators is established. In the last section V the possible tests of quark and field algebra models are presented.

2. Kinematic Analysis of Electroproduction Process with One Hadron Singled Out

We consider the deep inelastic scattering of lepton (electron or muon) on a proton

$$\ell + p \to \ell + a' + \text{"anything"} \tag{2.1}$$

in dependence of momentum and polarization of the scattered lepton and of momentum of the outgoing hadron a'. In the one photon approximation the kinematic conventions are shown in Fig.1:

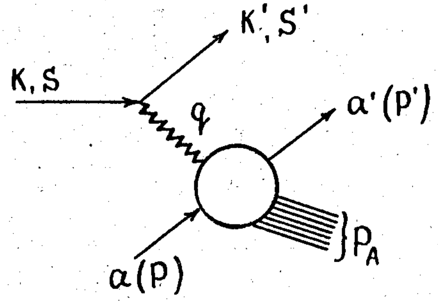

Fig. 1. Kinematics of lepton-hadron scattering. Here k, s (k', s') denote the four momentum and polarization vector of the ingoing (outgoing) lepton; p is the four momentum of the target hadron, p' the four momentum of the singled out hadron in final state. p_A denotes the total four momentum of the remaining hadrons; $q = k - k'$ is the four momentum of the virtual photon with $q^2 < 0$ in the physical region.

The T-matrix element of our process (2.1) is given by

$$T_{fi} = \frac{4\pi\alpha}{q^2} \, \bar{u}_{s'}(k')\gamma^{\mu} u_s(k) \langle p', p_A \text{ out} \,|\, J_{\mu}(0) \,|\, p \rangle , \qquad (2.2)$$

where $J_{\mu}(x)$ is the operator of the electromagnetic hadronic current. The differential cross section for the process when in the final state we detect the momentum and polarization of the outgoing lepton and the momentum of hadron a' can be written in standard fashion as

$$d\sigma = \frac{(4\pi\alpha)^2}{4\sqrt{(pk)^2 - m^2 m_\ell^2}} \cdot \frac{1}{q^4} \, \frac{1}{(2\pi)^6} \, \frac{d\vec{k}'}{2E'} \, \frac{d\vec{p}'}{2\mathcal{E}'} \, \bar{\rho}_{\mu\nu}(p, p', q) \, j^{\mu\nu}(k, k', s') \quad (2.3)$$

where

$$j^{\mu\nu}(k,k',s') = \frac{1}{2}\,\mathrm{Sp}\,[\gamma^{\mu}(\hat{k}+m_{\ell})\gamma^{\nu}(\hat{k}'+m_{\ell})\frac{1+\gamma_5\hat{s}'}{2}] =$$

$$= k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} + \frac{q^2}{2}g^{\mu\nu} + i\,m_{\ell}\,\epsilon^{\mu\nu\alpha\beta}q_{\alpha}s_{\beta}\,. \tag{2.4}$$

m is the proton mass and m_{ℓ} is the lepton mass. The second rank tensor $\bar{\rho}_{\mu\nu}$ is given by

$$\bar{\rho}_{\mu\nu}(p,p',q) = \bar{\sum_{A}}(2\pi)^4\,\delta(p+q-p'-p_A)\langle p\,(J_{\mu}(0)\,|\,p',p_A\,\mathrm{out}\rangle^{c}\langle p',p_A\,\mathrm{out}\,|\,J_{\nu}(0)|p\rangle$$

Here the superscript "c" denotes the omission of disconnected parts of the matrix elements.

From current conservation it follows that $q^{\mu}\bar{\rho}_{\mu\nu} = \bar{\rho}_{\mu\nu}q^{\nu} = 0$ and from hermiticity we obtain the condition that the real part of the tensor must be symmetric under exchange $\mu \leftrightarrow \nu$ and the imaginary part antisymmetric. These conditions, together with parity conservation, imply that $\bar{\rho}_{\mu\nu}$ can be expressed in the form

$$\bar{\rho}_{\mu\nu}(p,p',q) = \bar{\rho}_1\left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}\right) + \bar{\rho}_2 P_{\mu}P_{\nu} + \bar{\rho}_3 P'_{\mu}P'_{\nu} + \tag{2.6}$$

$$+\bar{\rho}_4(P_{\mu}P'_{\nu} + P_{\nu}P'_{\mu}) + i\,\bar{\rho}_5(P_{\mu}P'_{\nu} - P_{\nu}P'_{\mu}),$$

where

$$P_{\mu} = p_{\mu} - \frac{p\cdot q}{q^2}q_{\mu} \qquad P'_{\mu} = p'_{\mu} - \frac{p'\cdot q}{q^2}q_{\mu}\,. \tag{2.7}$$

The $\bar{\rho}_i$ ($i = 1,2,3,4,5$) are real functions depending on the four Lorentz-invariant variables which can be constructed from the four-vectors p, p' and q. A possible choice is the usual Mandelstam variables[x/]

$$\bar{s} = (p+q)^2 = m^2 + q^2 + 2\nu\,, \quad \nu = p.q$$
$$\bar{t} = (p'-q)^2 \equiv \Delta^2 \tag{2.8}$$
$$\bar{u} = (p'-p)^2 = m^2 + m'^2 - 2m\epsilon' \qquad \epsilon' = \frac{1}{m}pp'$$

and the square of the virtual photon mass q^2. Here m' is the mass of the final hadron $(p_0'^2 = m'^2)$. The usual condition

$$\bar{s} + \bar{t} + \bar{u} = m^2 + m'^2 + q^2 + m_A^2 \tag{2.9}$$

is fulfilled, where $m_A^2 = p_A^2$ is the square of the invariant mass of the outgoing hadron system ("anything"). The physical meaning of the additional variables ν and ϵ' is best seen in the laboratory system $(\vec{p} = 0)$. Then $\nu = m q_0$ is proportional to the energy of the virtual photon, $\epsilon' = p_0'$ is the energy of the outgoing hadron. It is useful also to introduce the invariant $\delta = -\frac{1}{m} p\Delta$ which is equal to $\delta = q_0 - p_0' = p_{A0} - m$ and thus represents the energy transfer in the laboratory frame.

Now it is easy to compute the invariant product of the two tensors

$$\bar{\rho}_{\mu\nu} j^{\mu\nu} = \bar{\rho}_1 2[(kk') - 2m_\ell^2] + \bar{\rho}_2 [2(pk)(pk') + m^2 \frac{q^2}{2}] +$$
$$+ \bar{\rho}_3 [2(p'k)(p'k') + m'^2 \frac{q^2}{2}] + \bar{\rho}_4 [2(pk)(p'k') + \tag{2.10}$$
$$+ 2(pk')(p'k) + (pp')q^2] - \bar{\rho}_5 2m_\ell \epsilon^{\mu\nu\alpha\beta} p_\mu p'_\nu q_\alpha s'_\beta .$$

x/ We use a bar to distinguish the Mandelstam variables of the crossed processes (see sec. III).

Using this relation and (2.3) we find the cross section in the laboratory frame

$$\frac{d\sigma}{d\Omega_{\vec{k}'}\, dE'\, d\Omega_{\vec{p}'}\, d\mathcal{E}'} = \frac{\alpha^2}{q^4} \cdot \frac{|\vec{k}'||\vec{p}'|}{|\vec{k}|} \cdot \frac{1}{64\ \pi^4 m} \cdot \times$$

$$\times \{ \bar{\rho}_1 2(EE' - |\vec{k}||\vec{k}'| \cos\theta - 2m_\ell^2) + \bar{\rho}_2 m^2(EE' + |\vec{k}||\vec{k}'| \cos\theta + m_\ell^2) +$$

$$+ \bar{\rho}_3 [2EE'\mathcal{E}'^2 - 2E\mathcal{E}'|\vec{p}'||\vec{k}'|\cos\psi' - 2E'\mathcal{E}'|\vec{p}'||\vec{k}|\cos\psi +$$

$$+ 2|\vec{p}'|^2|\vec{k}||\vec{k}'|\cos\psi\cos\psi' + m'^2(m_\ell^2 - EE' + |\vec{k}|\,|\vec{k}'|\cos\theta)] + \qquad (2.11)$$

$$+ \bar{\rho}_4\, 2m(EE'\mathcal{E}' - E|\vec{p}'||\vec{k}'|\cos\psi' - E'|\vec{p}'||\vec{k}|\cos\psi + \mathcal{E}|\vec{k}||\vec{k}'|\cos\theta + m_\ell^2\mathcal{E}') -$$

$$- \bar{\rho}_5\, 2mm_\ell|\vec{p}'|[s'_x(|\vec{k}|\sin\psi\sin\phi - |\vec{k}'|\sin\psi\sin\phi\cos\theta) +$$

$$+ s'_y(|\vec{k}'|\sin\psi\cos\phi\cos\theta - |\vec{k}'|\cos\psi\sin\theta - |\vec{k}|\sin\psi\cdot\cos\phi)$$

$$+ s'_z|\vec{k}'|\sin\psi\sin\phi\sin\theta]\}$$

with angles θ, ψ, ψ', ϕ defined according to fig.2 and with E, E', $\mathcal{E}'$ denoting the energies of the leptons and the final hadron, respectively.

Obviously, the structure function $\bar{\rho}_5$ can be found only by polarization experiments. If we detect the polarization of the ingoing lepton and sum over the polarization of the outgoing one we have the same formula (2.11) only with an additional factor 2 and with s' replaced by s (the spin vector of the ingoing lepton).

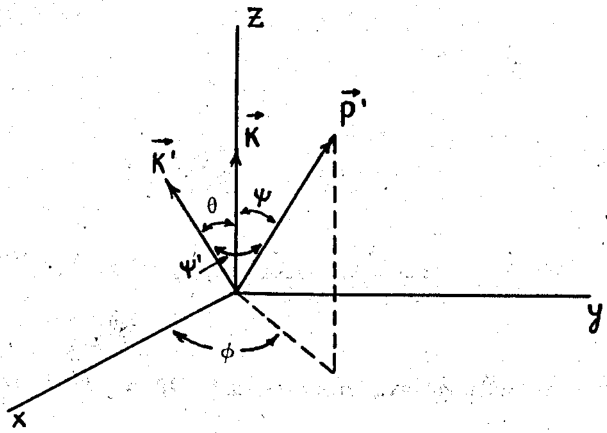

Fig.2. Definition of the angles in the laboratory frame. $\vec{k}'$ lies in the xz -plane, ψ' is the angle between $\vec{k}'$ and $\vec{p}'$. Note that the angles are connected by the relation $\cos\psi' = \cos\psi\cos\theta + \sin\psi\sin\theta\cos\phi$.

If we neglected the lepton mass $(m_\ell = 0)$ and the mass of the single outgoing hadron $(m' = 0)$ this formula greatly simplifies to

$$\frac{d\sigma}{d\Omega_{\vec{k}'}\, dE'\, d\Omega_{\vec{p}'}\, d\mathcal{E}'} = \frac{\alpha^2}{q^4} \cdot \frac{E'^2 \mathcal{E}'}{32\pi^4 m} \Big[\bar{\rho}_1\, 2\sin^2\frac{\theta}{2} + \bar{\rho}_2\, m^2 \cos^2\frac{\theta}{2} + \tag{2.12}$$

$$+ \bar{\rho}_3\, 4\mathcal{E}'^2 \sin^2\frac{\psi}{2} \sin^2\frac{\psi}{2} + \bar{\rho}_4\, m\mathcal{E}' (1 + \cos\theta - \cos\psi - \cos\psi')\Big]$$

with

$$q^2 = (k-k')^2 = 2m_\ell^2 - 2EE' + 2|\vec{k}||\vec{k}'|\cos\theta \approx -4EE'\sin^2\frac{\theta}{2} ,$$

Comparison with the corresponding formulae of ordinary electroproduction (see, e.g., /5/) is straightforward.

Another form for the cross section is obtained if we expand the hadronic tensor $\bar{\rho}_{\mu\nu}$ by means of polarization vectors of the virtual photon, as has been done for the crossed process of lepton pair production in /1/.

The longitudinal and the two transverse polarization vectors are defined by

$$\epsilon_\mu^{(L)} = \frac{1}{\sqrt{-P^2}} P_\mu$$

$$\epsilon_\mu^{(T_1)} = \sqrt{\frac{P^2}{(PP')^2 - P^2P'^2}}\left(P'_\mu - \frac{PP'}{P^2} P_\mu\right) \qquad (2.13)$$

$$\epsilon_\mu^{(T_2)} = \frac{1}{\sqrt{q^2}} \frac{1}{\sqrt{(pp')^2 - m^2m'^2}} \epsilon_{\mu\nu\lambda\rho}\, p^\nu p'^\lambda q^\rho .$$

These vectors are orthogonal, normalized to -1 and they fulfill the conditions

$$\epsilon_\mu^{(a)} q^\mu = 0 \qquad a = T_1, T_2, L \qquad (2.14)$$

$$\sum_{a=T_1,T_2,L} \epsilon_\mu^{(a)} \epsilon_\nu^{(a)} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} . \qquad (2.15)$$

Using these polarization vectors we can represent $\bar{\rho}_{\mu\nu}$ in the form

$$\bar{\rho}_{\mu\nu}(p,p',q) = \bar{\rho}_{T_1} \epsilon_\mu^{(T_1)} \epsilon_\nu^{(T_1)} + \bar{\rho}_{T_2} \epsilon_\mu^{(T_2)} \epsilon_\nu^{(T_2)} + \bar{\rho}_L \epsilon_\mu^{(L)} \epsilon_\nu^{(L)} + \qquad (2.16)$$
$$+ \bar{\rho}_{TL}^{(+)} (\epsilon_\mu^{(T_1)} \epsilon_\nu^{(L)} + \epsilon_\nu^{(T_1)} \epsilon_\mu^{(L)}) + i \bar{\rho}_{TL}^{(-)} (\epsilon_\mu^{(T_1)} \epsilon_\nu^{(L)} - \epsilon_\nu^{(T_1)} \epsilon_\mu^{(L)}),$$

where the structure functions $\bar{\rho}_a$ $(a = T_1, T_2, L, TL^{(+)}, TL^{(-)})$ (which, of course, also depend on $q^2, \bar{s}, \bar{t}, \bar{u}$) are connected with our former structure functions $\bar{\rho}_i$ ($i = 1,2,3,4,5$) by

$$\bar{\rho}_1 = \bar{\rho}_{T_2}$$

$$\bar{\rho}_2 = \frac{1}{P^2}\left[\bar{\rho}_{T_2} - \bar{\rho}_L - \frac{(PP')^2}{P^2P'^2 - (PP')^2}(\bar{\rho}_{T_1} - \bar{\rho}_{T_2}) - \frac{2PP'}{\sqrt{P^2P'^2 - (PP')^2}} \bar{\rho}_{TL}^{(+)}\right]$$

$$\bar{\rho}_3 = \frac{P^2}{(PP')^2 - P\,P'^2}(\bar{\rho}_{T_1} - \bar{\rho}_{T_2})$$

$$\bar{\rho}_4 = \frac{1}{\sqrt{P^2P'^2-(PP')^2}}\bar{\rho}^{(+)}_{TL} + \frac{PP'}{P^2P'^2-(PP')^2}(\bar{\rho}_{T_1} - \bar{\rho}_{T_2}) \qquad (2.17)$$

$$\bar{\rho}_5 = \frac{1}{\sqrt{P^2P'^2-(PP')^2}}\rho^{(-)}_{TL} .$$

The formfactors $\bar{\rho}_i$ ($i = 1,2,3,4,5$) are suitable from the experimental point of view, while the structure functions $\bar{\rho}_a$ ($a = T_1, T_2, L, TL^{(+)}, TL^{(-)}$) are more convenient for theoretical considerations (see section IV).

III. Connection with Lepton-Pair Production

As has already been mentioned, the crossed (u -channel) process to that which we consider here is the process of lepton-pair production in deep inelastic hadron-hadron collision

$$\bar{a}' + a \rightarrow \underset{\ell^+\ \ell^-}{\gamma^*} + \text{"anything"} \qquad (3.1)$$

which is shown in fig. 3.

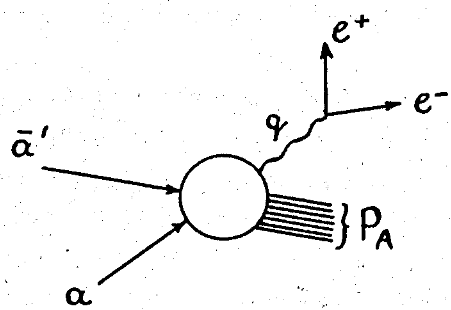

Fig.3. Kinematics of lepton-pair production. In this case q^2 is time like ($q^2>0$) in the physical region.

This process has been theoretically studied in [1] and some preliminary experimental data were reported in [2]. The hadronic part of the process is described by the tensor

$$\rho_{\mu\nu}(p,p',q) = \sum_A (2\pi)^4 \delta(p+p'-q-p_A) \langle pp' \text{ in} | J_\mu(0) | A \text{ out} \rangle^c \langle A \text{ out} | J_\nu(0) | p,p' \text{ in} \rangle^c$$

which can be expanded in analogy with (2.6) in the form

$$\rho_{\mu\nu} = \rho_1 \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \rho_2 P_\mu P_\nu + \rho_3 P'_\mu P'_\nu + \rho_4 (P_\mu P'_\nu + P_\nu P'_\mu) + i\rho_5 (P_\mu P'_\nu - P_\nu P'_\mu) = \tag{3.3a}$$

$$= \rho_{T_1} \epsilon^{(T_1)}_\mu \epsilon^{(T_1)}_\nu + \rho_{T_2} \epsilon^{(T_2)}_\mu \epsilon^{(T_2)}_\nu + \rho_L \epsilon^{(L)}_\mu \epsilon^{(L)}_\nu + \rho^{(+)}_{TL} (\epsilon^{(T_1)}_\mu \epsilon^{(L)}_\nu + \epsilon^{(T_1)}_\nu \epsilon^{(L)}_\mu) + \tag{3.3b}$$
$$+ i\rho^{(-)}_{TL} (\epsilon^{(T_1)}_\mu \epsilon^{(L)}_\nu - \epsilon^{(T_1)}_\nu \epsilon^{(L)}_\mu) .$$

The structure functions ρ_i ($i = 1,2,3,4,5$) and ρ_a ($a = T_1, T_2, L, TL^{(+)}, TL^{(+)}$) depend again on the four linearly independent Lorentz scalars, constructed from p, p' and q. The Mandelstam variables here have the form

$$s = (p+p')^2 \quad , t = (p'-q)^2 \quad u = (p-q)^2 . \tag{3.4}$$

Ordinary reduction techniques yield the following connection between matrix elements

$$\langle a(p) \text{ in} | J_\mu(0) | a'(p'), A \text{ out} \rangle^c = \langle \text{in } a(p) \bar{a}'(-p) | J_\mu(0) | A \text{ out} \rangle^c . \tag{3.5}$$

We want to remark that $\bar{a}'(-p')$ represents the antiparticle to a' with momentum $-\vec{p}'$ and with unphysical energy $-p_0'$ ($p_0' > 0$). Therefore the right-hand side of eq. (3.5) has no physical meaning. It can only be understood as the analytic continuation of the function $\lim_{t\to-\infty} \int d\vec{x}\, f^*_p(x) \overleftrightarrow{\partial}_0 \cdot \langle a(p) | \Phi^*(x) J_\mu(0) | A \text{out} \rangle^c$ (which has a physical meaning only for $p_0' > 0$ representing then the matrix element $\langle \text{in } a(p) \bar{a}'(p') | J_\mu(0) | A \text{ out} \rangle^c$) to negative values of p_0' .

Using eq. (3.5) we get the crossing relation

$$\bar{\rho}_{\mu\nu}(p,p'q) = \rho_{\mu\nu}(p,-p',-q) . \tag{3.6}$$

This implies the following relation between the structure functions

$$\bar{\rho}_i(q^2, pp', pq, p'q) = \rho_i(q^2, -pp', -pq, p'q) \quad i = 1,2,3 \text{ or } T_1, T_2, L$$

$$\bar{\rho}_i(q^2, pp', pq, p'q) = -\rho_i(q^2, -pp', -pq, p'q) \quad i = 4,5 \text{ or } TL^{(\pm)} . \tag{3.7}$$

Expressed in Mandelstam variables this reads:

$$\bar{\rho}_i(q^2, \bar{s}, \bar{t}, \bar{u}) = \rho_i(q^2, \bar{u}, \bar{t}, \bar{s}) \quad i = 1,2,3 \text{ or } T_1, T_2, L$$

$$\bar{\rho}_i(q^2, \bar{s}, \bar{t}, \bar{u}) = -\rho_i(q^2, \bar{u}, \bar{t}, \bar{s}) \quad i = 4,5 \text{ or } TL^{(\pm)} \tag{3.8}$$

IV. Current Commutators and Structure Functions in the High Energy Limit

Let us now consider the expression

$$\bar{R}_{\mu\nu}(p, p', q) = \int dx\, e^{iqx} \langle a(p)\bar{a}'(-p') \text{in} | [J_\mu(x), J_\nu(0)] | a(p), \bar{a}'(-p') \text{in} \rangle^{c} =$$

$$= \bar{r}_{\mu\nu}(p, p', q) - \bar{r}_{\nu\mu}(p, p', -q) \tag{4.1}$$

and divide $\bar{r}_{\mu\nu}$ into connected and disconnected parts in the following way

$$\bar{r}_{\mu\nu}(p, p', q) = \int dx e^{iqx} \langle a(p)\bar{a}'(-p') \text{in} | J_\mu(x) J_\nu(0) | a(p)\bar{a}'(-p') \text{in} \rangle^{c} =$$

$$= \sum_A^{c} (2\pi)^4 \delta(p + q - p' - p_A) \langle a(p)\bar{a}'(-p') \text{in} | J_\mu(0) | A \text{out} \rangle \langle A \text{out} | J_\nu(0) | a(p)\bar{a}'(-p') \text{in} \rangle =$$

$$= \sum_A (2\pi)^4 \delta(p + q - p' - p_A) \langle a(p)\bar{a}'(-p') \text{in} | J_\mu(0) | A \text{out} \rangle^{c} \langle A \text{out} | J_\nu(0) | a(p)\bar{a}'(-p') \text{in} \rangle^{c} + \tag{4.2}$$

$$+ \text{weakly connected parts} = \bar{\rho}_{\mu\nu}(p, p', q) + \bar{\bar{\rho}}_{\mu\nu}(p, p', q),$$

where the superscript "c" in the sum denotes that we disregard the completely disconnected parts. $\bar{\rho}_{\mu\nu}$ is the completely connected part involved in the definition of the cross section and defined by (2.6); $\bar{\bar{\rho}}_{\mu\nu}$ represents the weakly connected parts which correspond to the contribution of the so-called z -diagrams and which we neglect here. The omitting of these weakly connected parts foots on the same reasons as in /1/.

We now use the unphysical rest frame of the virtual photon where $\vec{q}=0$. In this frame only space components of the gauge invariant tensors are nonzero and therefore the decompositions (2.6) and (2.11) take the form x/

$$\bar{\rho}_{ij}=\bar{\rho}_1\delta_{ij}+\bar{\rho}_2 p_i p_j+\bar{\rho}_3 p_i' p_j'+\bar{\rho}_4(p_i p_j'+p_j p_i')+i\bar{\rho}_5(p_i p_j'-p_j p_i')= \tag{4.3}$$

$$=\bar{\rho}_{T_2}\delta_{ix}\delta_{jx}+\rho_{T_2}\delta_{iy}\delta_{yj}+\bar{\rho}_L\delta_{iz}\delta_{jz}+\bar{\rho}^{(+)}_{TL}(\delta_{ix}\delta_{jz}+\delta_{jx}\delta_{iz})+$$

$$+i\bar{\rho}^{(-)}_{TL}(\delta_{ix}\delta_{jz}-\delta_{jx}\delta_{iz}).$$

It is obvious that the quantities $\bar{R}_{ij}$, $\bar{r}_{ij}$ and $\bar{\bar{\rho}}_{ij}$ can be expanded in a similar way

$$\bar{R}_{ij}=\bar{R}_1\delta_{ij}+\ldots=\bar{R}_{T_1}\delta_{ix}\delta_{jx}+\ldots$$

$$\bar{r}_{ij}=\bar{r}_1\delta_{ij}+\ldots=\bar{r}_{T_1}\delta_{ix}\delta_{jx}+\ldots \tag{4.4}$$

$$\bar{\bar{\rho}}_{ij}=\bar{\bar{\rho}}_1\delta_{ij}+\ldots=\bar{\bar{\rho}}_{T_1}\delta_{ix}\delta_{jx}+\ldots$$

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x/ In the following we choose the space parts of the polarization vectors as the coordinate axis.

The structure functions defined here are connected by the relations

$$\bar{R}_a(\vec{p},\vec{p}\,';q_0) = \bar{r}_a(\vec{p},\vec{p}\,';q_0) - \bar{r}_a(\vec{p},\vec{p}\,';-q_0) =$$

$$= \epsilon(q_0)\bar{\rho}_a(\vec{p},\vec{p}\,';|q_0|) + \epsilon(-q_0)\bar{\bar{\rho}}_a(\vec{p},\vec{p}\,';-|q_0|) \quad \text{for } a = T_1, T_2, L, TL^{(+)} \tag{4.5}$$

and

$$\bar{R}^{(-)}_{TL}(\vec{p},\vec{p}\,';q_0) = \bar{r}^{(-)}_{TL}(\vec{p},\vec{p}\,',q_0) + \bar{r}^{(-)}_{TL}(\vec{p},\vec{p}\,';-q_0) =$$

$$= \bar{\rho}^{(-)}_{TL}(\vec{p},\vec{p}\,',|q_0|) + \bar{\bar{\rho}}^{(-)}_{TL}(\vec{p},\vec{p}\,';-|q_0|). \tag{4.6}$$

We see that $\bar{R}_{T_1}$, $\bar{R}_{T_2}$, $\bar{R}_L$, $\bar{R}^{(+)}_{TL}$ are odd functions of q_0 and $\bar{R}^{(-)}_{TL}$ is an even one.

Integrating (4.1) over dq_0 and $q_0\,dq_0$, respectively, we obtain the following relations

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dq_0\,\bar{R}_{ij}(\vec{p},\vec{p}\,';q_0) = i\bar{B}_{ij}(\vec{p},\vec{p}\,') \tag{4.7}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} q_0\,dq_0\,\bar{R}_{ij}(\vec{p},\vec{p}\,';q_0) = \bar{C}_{ij}(\vec{p},\vec{p}\,'), \tag{4.8}$$

where

$$\bar{B}_{ij}(\vec{p},\vec{p}\,') = -i\int d\vec{x} < a(p)\,a'(-p')\,in\,|[J_i(\vec{x},0), J_j(0)]|\,a(p)\bar{a}'(-p')in> \tag{4.9}$$

$$\bar{C}_{ij}(\vec{p},\vec{p}\,') = -i\int d\vec{x} < a(p)\bar{a}'(-p')\,in\,|[\dot{J}_i(\vec{x},0), J_j(0)]|\,a(p)\bar{a}'(-p')in> \tag{4.10}$$

The relations (4.7) and (4.8) are equivalent to the following ones

$$\frac{2}{\pi}\int_0^{\infty} dq_0\,\bar{R}^{(-)}_{TL}(\vec{p},\vec{p}\,';q_0) = \bar{B}_{xz}(\vec{p},\vec{p}\,') - \bar{B}_{zx}(\vec{p},\vec{p}\,') \tag{4.11a}$$

$$\frac{1}{\pi}\int_0^{\infty} dq_0\,q_0\,\bar{R}_{T_1}(\vec{p},\vec{p}\,';q_0) = \bar{C}_{xx}(\vec{p},\vec{p}\,') \tag{4.11b}$$

$$\frac{1}{\pi}\int_0^{\infty} dq_0\,q_0\,\bar{R}_{T_2}(\vec{p},\vec{p}\,';q_0) = \bar{C}_{yy}(\vec{p},\vec{p}\,') \tag{4.11c}$$

$$\frac{1}{\pi}\int_0^\infty dq_0 q_0 \bar{R}_L(\vec{p},\vec{p}\,';q_0) = \bar{C}_{zz}(\vec{p},\vec{p}\,') \tag{4.11d}$$

$$\frac{2}{\pi}\int_0^\infty dq_0 q_0 \bar{R}^{(+)}_{TL}(\vec{p},\vec{p}\,';q_0) = \bar{C}_{xz}(\vec{p},\vec{p}\,') + \bar{C}_{zx}(\vec{p},\vec{p}\,') \tag{4.11e}$$

Now we take the limit $p_0, -p_0' \to \infty$ assuming $\alpha = \frac{p_0'}{p_0}$ = fixed and $\beta = \frac{p_z'}{p_0}$ = fixed. It is reasonable to expect that the following dimensionless quantities exist

$$\bar{B}_{ij}(\alpha,\beta) = \lim_{\substack{p_0, -p_0' \to \infty \\ \alpha,\beta = \text{fixed}}} p_0 \bar{B}_{ij}(\vec{p},\vec{p}\,') \tag{4.12}$$

$$\bar{C}_{ij}(\alpha,\beta) = \lim_{\substack{p_0, -p_0' \to \infty \\ \alpha,\beta = \text{fixed}}} \bar{C}_{ij}(\vec{p},\vec{p}\,'). \tag{4.13}$$

For the study of the high energy behaviour of the structure functions we use also the variable $\bar{\omega} = -\frac{q^2}{2\nu}$ which here is equal to $-\frac{q_0}{2p_0}$ x/. The high energy limit of our structure functions are found by that of the lepton pair production structure functions. In /1/ and /8/ it is shown that these behave like

$$\rho_a(s, q^2, \alpha, \nu) \to \frac{\omega^2}{q^2} F_a(\alpha,\beta,\omega) \quad a = T_1, T_2, L, TL^{(+)}, TL^{(-)}$$

when $s, q^2, \nu \to \infty$ at fixed ω and $\frac{s}{2\nu}$. This limit procedure is equivalent to the case of p_0, $p_0' \to \infty$ at fixed α and β. We now assume that these limits also exist for spacelike q^2 i.e. $q^2 \to -\infty$) and for negative α, p_0' and ν. Then from crossing relation in the form

$$\bar{\rho}_a(\bar{u}, q^2, \alpha, \nu) = \pm \rho_a(s, q^2, -\alpha-\nu)$$

x/ We remember that the physical region of $\bar{\omega}$ is determined by $0 < \bar{\omega} < 1$ in the case of usual electroproduction.

we get the behaviour

$$\lim_{\substack{\bar{u}\to\infty \\ \nu,\, q^2\to-\infty \\ \omega,\, \frac{s}{2\nu}\approx \text{fixed}}} \bar{\rho}_a(\bar{u}, q^2, \alpha, \nu) = \frac{\bar{\omega}^2}{q^2} F_a(-\alpha, \beta, \bar{\omega}) \quad a = T_1, T_2, L. \tag{4.14a}$$

and

$$\lim_{\substack{\bar{u}\to\infty \\ \nu,\, q^2\to-\infty \\ \omega,\, \frac{s}{2\nu}\approx \text{fixed}}} \bar{\rho}_b(\bar{u}, q^2, \alpha, \nu) = -\frac{\bar{\omega}^2}{q^2} F_b(-\alpha, \beta, \bar{\omega}) \quad b = TL^{(\pm)} \tag{4.14b}$$

Thereby $F(-\alpha, \beta, \bar{\omega})$ must be understood as the analytic continuation of the functions $F(\alpha, \beta, \omega)$ to the case of $q^2<0$ and $-p_0'<0$,i.e. to values of $\alpha<0$ .If we now apply these limits to (4.11) and substitute the variable $\omega \to -\frac{q_0}{2p_0}$ we get the following sum rules

$$\frac{1}{\pi}\int_0^{\omega^*} d\bar{\omega}\, F_{TL}^{(-)}(-\alpha, \beta, \bar{\omega}) = \bar{B}_{xz}(\alpha, \beta) - \bar{B}_{zx}(\alpha, \beta) \tag{4.15a}$$

$$\frac{1}{\pi}\int_0^{\omega^*} d\bar{\omega}\, \bar{\omega}\, F_{T_1}(-\alpha, \beta, \omega) = \bar{C}_{xx}(\alpha, \beta) \tag{4.15b}$$

$$\frac{1}{\pi}\int_0^{\omega^*} d\bar{\omega}\, \bar{\omega}\, F_{T_2}(-\alpha, \beta, \bar{\omega}) = \bar{C}_{yy}(\alpha, \beta) \tag{4.15c}$$

$$\frac{1}{\pi}\int_0^{\omega^*} d\bar{\omega}\, \bar{\omega}\, F_L(-\alpha, \beta, \bar{\omega}) = \bar{C}_{zz}(\alpha, \beta) \tag{4.15d}$$

$$-\frac{2}{\pi}\int_0^{\omega^*} d\bar{\omega}\, \bar{\omega}\, F_{TL}^{(+)}(-\alpha, \beta, \bar{\omega}) = \bar{C}_{xz}(\alpha, \beta) + \bar{C}_{zx}(\alpha, \beta) \tag{4.15e}$$

where $\omega^* = -\frac{p_0' - p_0}{2p_0} = \frac{1}{2}(1-\alpha)$.

V. The Test of Models

Equal time commutators between space components of the electromagnetic current, which appear in the former section, are highly model dependent, and therefore can provide a test for different models.

As it is well known, the "gluon" quark model and the algebra of fields, respectively, give the following values for the commutators

$$[J_i(\vec{x},0), J_j(0)] = \begin{cases} 2i\delta(\vec{x})\epsilon_{ijk}\bar{\psi}\gamma_5\gamma^k Q^2\psi & \text{(quark model)} \quad (5.1a) \\ 0 & \text{(algebra of fields)} \quad (5.1b) \end{cases}$$

$$[\dot{J}_i(\vec{x},0), J_j(0)] = \begin{cases} -\delta(\vec{x})\bar{\psi}\{i(\gamma_i\partial_j + \gamma_j\partial_i - 2\vec{\gamma}\vec{\partial}\delta_{ij}) - \\ \quad - 2g(\gamma_i B_j + \gamma_j B_i - 2\vec{\gamma}\vec{B}\delta_{ij}) + 4M\delta_{ij}\}Q^2\psi \quad (5.2a) \\ \quad \text{(quark model)} \\ \delta(x)C_{ab}J^a_i(0)J^b_j(0) + \text{C-numbers (algebra of fields)}. \quad (5.2b) \end{cases}$$

From field algebra we only can derive that in (4.15a) $\int d\bar{\omega}\, F^{(-)}_{TL} = 0$.

Let us now consider the case of the quark model. The currents which appear on the right hand sides of (5.1) and (5.2) can be written in this model schematically (we omit quark- and spin indices for simplicity) as

$$\begin{aligned} \bar{\psi}(x)\Gamma^k\psi(x) = \int\int \frac{d\vec{p}\,'\,d\vec{p}\,''}{(2\pi)^3\sqrt{2p_0' 2p_0''}} \{ & a^+(\vec{p}\,'')a(\vec{p}\,')\bar{u}(\vec{p}\,'')\Gamma^k u(\vec{p}\,')e^{i(p''-p')x} + \\ & + b(\vec{p}\,'')b^+(p')\bar{v}(\vec{p}\,'')\Gamma^k v(\vec{p}\,')e^{-i(p''-p')x} + \\ & + a^+(\vec{p}\,'')b^+(\vec{p}\,')\bar{u}(\vec{p}\,'')\Gamma^k\gamma_5 u(\vec{p}\,')e^{i(p''+p')x} + \\ & + b(\vec{p}\,'')a(\vec{p}\,')\bar{v}(\vec{p}\,'')\Gamma^k\gamma_5 v(\vec{p}\,')e^{-i(p''+p')x}\} \end{aligned} \quad (5.3)$$

Γ^k stands for γ^k, $\gamma^k\gamma_5$ and 1, respectively. The last two terms between the brackets create (or annihilate) only quark-antiquark pairs and thus contribute to the z-diagrams, so that we can neglect them.

For the construction of the ingoing states in the quark picture we suppose that we can choose the so-called "model state" (i.e. the three- or two quark bound state without quark-antiquark pair excitation). This simplification is probably not too bad because all the additional parts contain quark-antiquark pairs and thus also contribute as z-diagrams.

For example, the pion + proton state can be represented in this idealized model as

$$|P(\vec{p}), \pi(\vec{p}\,')> = \sum_{\vec{k}_1,\vec{k}_2,\vec{k}_3} \sum_{\vec{K}_1,\vec{K}_2} f^{(P)}(\vec{p}\,;\vec{k}_1,\vec{k}_2,\vec{k}_3)\, f^{(\pi)}(\vec{p}\,';\vec{K}_1,\vec{K}_2)\, a_p^+(\vec{k}_1) \times \tag{5.4}$$

$$\times\, a_p^+(\vec{k}_2) a_n^+(\vec{k}_3) a_p^+(\vec{K}_1) b_n^+(\vec{K}_2)|0> \quad .$$

where the binding functions $f^{(P)}$ and $f^{(\pi)}$ contain appropriate δ-functions expressing energy momentum conservation.

We now sandwich our current (5.3) with these states and use the known anticommutation relations of creation and annihilation operators of the quark. With the help of the relations

$$\begin{aligned}
\bar{u}_{s'}(\vec{p})\, u_s(\vec{p}) &= \frac{M}{p_0}\,\delta_{ss'}\,, \\
\bar{u}_{s'}(\vec{p})\gamma^k u_s(\vec{p}) &= \frac{p^k}{p_0}\,\delta_{ss'}\,, \\
\bar{u}_{s'}(\vec{p})\gamma^k\gamma_5 u_s(\vec{p}) &= \frac{p^k}{p_0}\,\frac{(\vec{\sigma}\cdot\vec{p})_{ss'}}{p_0+M} + \frac{M}{p_0}(\sigma^k)_{ss'}\,,
\end{aligned} \tag{5.5}$$

we get after some algebra in our limit the following relations

$$B_{xz}(\alpha,\beta) = -B_{zx}(\alpha,\beta) = \text{const} \tag{5.6}$$

$$C_{xx}(\alpha,\beta) = C_{yy}(\alpha,\beta) = \text{const} \tag{5.7}$$

$$C_{zz}(\alpha,\beta) = 0\,. \tag{5.8}$$

Thereby we have made the additional assumption that in our limit (p_z , $-p_z' \to \infty$; $p_x\ p_y\ p_x'\ p_y'$ finite) the states are dominated by quark configurations with great z -components of the single quark momenta.

From (4.15a) and (5.1) it follows that quark model and gauge field model can be distinguished in principle by means of polarization experiments because of

$$\int_0^{\omega^*} d\,\bar{\omega}\, F_{TL}^{(-)}(-\alpha,\beta,\bar{\omega}) = \begin{cases} \text{const (quark model)} \\ 0 \quad \text{(field algebra)} \end{cases} \tag{5.9}$$

From (5.7) and (5.8) we see that quark model predicts a large contribution from transversally polarized virtual photons, in accordance with the conclusion of Callan and Gross/6/ for the case of usual electroproduction.

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