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**ABSENCE OF LONG-RANGE ORDER
IN QUANTUM FERROELECTRICS
WITH CORRELATED RANDOM FIELDS**

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The equivalence, about six dimensions, of the critical behaviour of n -component systems in uncorrelated random fields and of the corresponding pure classical ones in two dimensions less has been known for some time^{/1-3/}. An analogous result has been obtained by perturbative renormalization group (RG) approaches for random field (RF) quantum systems at temperature $T=0$. In particular, the dimensional reductions $d \rightarrow d-3$, $d \rightarrow d-4$ and $d - (2+z)$ (z is the zero temperature dynamical critical exponent) have been found for transverse Ising model^{/4/}, bosonized systems^{/5/} and nonbosonized systems^{/6/} respectively, in the presence of uncorrelated RFs. However, the above equivalence or its lack in lower dimensions up to the unknown lower critical one d_{CL} is at present controversial. Very recently, most of the controversy has focused on the RF Ising model due to its experimental relevance^{/7-9/}. For systems with continuous symmetry ($n \geq 2$ where n is the order parameter dimension), both perturbative dimensional reduction^{/1-6/} and domain arguments^{/4,10/} yield $d_{CL} = 4$. Temptation to prove this result rigorously has been made by Schuster^{/11/} for RF classical systems. However, his demonstration cannot be considered rigorously due to the use of the replica trick^{/12/}.

Recent investigations of RF classical systems^{/13,14/} indicate that the conventional perturbative methods may lead to misleading conclusions since nonperturbative effects (e.g., Griffiths singularities) become important and probably destroy dimensional reduction at least near the lower critical dimensionality. Analogous considerations are true when random anisotropies are present^{/14/}.

Such a classical scenario opens questions also for RF quantum systems. The previous perturbative RG predictions of the ($T=0$)-dimensional reduction cannot be naively extrapolated in lower dimensions so that they do not give reliable estimates for d_{CL} . On the other hand, for d -dimensional random quantum systems at $T=0$ which, when pure, are equivalent to the corresponding pure classical systems in $d+1$ dimensions, an estimate of d_{CL} may be made by using domain-like arguments^{/4/}. Nevertheless, for general quantum systems^{/15,16/}, the above-mentioned equivalence in absence of impurities is broken down and a direct extension of classical domain arguments seems to be no more possible. The situation appears more dramatic where correlated RFs and long-range interactions are involved. In this case, an explicit nonperturbative calculation of d_{CL} has been made only for

RF bosonized systems in the large- n limit^{/17/} and at the present only RG information on the upper critical dimension in bosonized^{/18/} and nonbosonized^{/19/} quantum systems with correlated RGs is available.

Since rigorous methods for calculating d_{CL} for RF systems are lacking, the aim of this paper is to give a trustworthy estimate of d_{CL} for a class of quantum systems with $n > 2$ in the presence of long-range interactions and correlated RFs. This will be realized by using a quantum extension of the Rice method^{/20/} for the destruction of long-range order in pure classical systems with continuous symmetry. This method has been already used for a calculation of d_{CL} for the classical random axis model^{/21/} and in connection with the destruction of long-range order of the charge-density-wave state in a Peierls transition when uncorrelated quenched impurities are present^{/22/}. Here we consider the case $n = 2$ only, being a straightforward extension to the general case $n \geq 2$.

For concreteness reasons, we refer to a quantum model appropriate to the description of RF quantum ferroelectrics, as $KTaO_3$ or $SrTiO_3$ with Li quenched impurities for which some experimental results are available^{/23/}, and of other quantum systems which show displacive structural phase transitions^{/24/}. However, the extended Rice method, although nonrigorous, is sufficiently general and physically quite reliable to yield, with light modifications, an estimate of d_{CL} for other RF quantum systems^{/5,6,18,19/}.

The mentioned ($n = 2$)-quantum systems in the presence of RFs can be conveniently described by means of the generalized quantum Ginzburg-Landau-Wilson (GLW) functional:

$$\mathcal{K}\{\psi, h\} = \int d^d \mathbf{x} \int_0^{1/T} d\tau \{ c |\nabla^{\sigma/2} \psi(\vec{x}, \tau)|^2 + r |\psi(\vec{x}, \tau)|^2 + \left| \frac{\partial \psi(\vec{x}, \tau)}{\partial \tau} \right|^2 + \frac{u}{2} |\psi(\vec{x}, \tau)|^4 - \frac{1}{2} [h(\vec{x}) \psi^*(\vec{x}, \tau) + h^*(\vec{x}) \psi(\vec{x}, \tau)] \}, \quad (1)$$

where the coupling parameters c, r, u depend on the system under study^{/23-25/}, $0 < \sigma \leq 2$ for including also long-range interactions, $\psi(\vec{x}, \tau) = \left(\frac{T}{V}\right)^{1/2} \sum_q e^{i(\vec{k}\vec{x} - \omega_\ell \tau)} \psi(q)$ is a com-

plex order parameter field with $0 \leq r \leq 1/T$, $q \equiv (\vec{k}, \omega_\ell)$, $\omega_\ell = 2\pi \ell T$ ($\ell = 0, \pm 1, \pm 2, \dots$), V is the volume of the system and Λ is a wave-vector cut off. In (1), the complex random field $h(\vec{x}) = h_1(\vec{x}) + ih_2(\vec{x})$ is governed by a Gaussian probability

distribution $\mathcal{P}\{h\}$ with Fourier component averages:

$$[h^*(\vec{k})]_{av} = [h(\vec{k})]_{av} = 0, \quad [h^*(\vec{k}) h(\vec{k}')]_{av} = \delta_{\vec{k}\vec{k}'} g(\vec{k}), \quad (2)$$

where^{/19/}:

$$g(\vec{k}) \approx \Delta_1 + \Delta_2 |\vec{k}|^\theta \quad (3)$$

for small $|\vec{k}|$ and arbitrary θ . Notice that, when $\theta \geq 0$ with $\Delta_1 \neq 0$ ($i = 1, 2$) or when $\Delta_2 = 0$, the short-range correlated RF case will be reproduced with $\Delta = \Delta_1 + \Delta_2$ or $\Delta = \Delta_1$ respectively.

We are interested to calculate the random two-point propagator:

$$G(\vec{x}, \tau) = [G(\vec{x}, \tau; \{h\})]_{av} = [\langle \psi(\vec{x}, \tau) \psi^*(0, 0) \rangle]_{av} \quad (4)$$

in the limit as $|\vec{x}| \rightarrow \infty$, keeping in mind that, as a general criterion for the existence of long-range order, $G(\vec{x}, \tau)$ must be a finite constant in the limit as the separation $|\vec{x}|$ goes to infinity^{/26, 27/}.

We start by assuming that, for some values of the parameter r driving the phase transitions, the "pure" quantum system has a long-range order both at $T \neq 0$ (for $d > \sigma$) and $T = 0$ (for $d > \sigma/2$) and the order parameter is given by $\psi = \psi_0 e^{i\phi_0}$, where $\psi_0 > 0$ (with $\psi_0^2 \approx -r/u$) and ϕ_0 is an arbitrary constant. Then, $G(\vec{x}, \tau)$ can be evaluated in terms of the approximate GLW functional obtained from (1) by expanding $\psi(\vec{x}, \tau)$ about ψ_0 and keeping only quadratic terms in the involved small quantities. This is quite justified under condition of weak disorder which is assumed here "a priori". Writing $\psi(\vec{x}, \tau)$ in terms of its modulus and phase as $\psi(\vec{x}, \tau) = (\psi_0 + \tilde{\psi}(\vec{x}, \tau)) e^{i\phi(\vec{x}, \tau)}$, to second order in real small quantities, a simple algebra yields the approximate random quantum functional:

$$\mathcal{K}\{\psi, h\} \approx K + \mathcal{K}\{\tilde{\psi}, h_1\} + \mathcal{K}\{\phi, h_2\}, \quad (5)$$

where the constant K is unessential and:

$$\mathcal{K}\{\tilde{\psi}, h_1\} = \int d^d \mathbf{x} \int_0^{1/T} d\tau \{ c (\nabla^{\sigma/2} \tilde{\psi}(\vec{x}, \tau))^2 + \left(\frac{\partial \tilde{\psi}(\vec{x}, \tau)}{\partial \tau} \right)^2 - 2r \tilde{\psi}^2(\vec{x}, \tau) - h_1(\vec{x}) \tilde{\psi}(\vec{x}, \tau) \}, \quad (6)$$

$$\mathcal{H}\{\phi, h_2\} = \int d^d \mathbf{x} \int_0^{1/T} dr \left\{ c \psi_0^2 (\nabla^{\sigma/2} \phi(\vec{x}, r))^2 + \psi_0^2 \left(\frac{\partial \phi(\vec{x}, r)}{\partial r} \right)^2 - \psi_0 h_2(\vec{x}) \phi(\vec{x}, r) \right\}. \quad (7)$$

In (6)-(7) the real quantities $\tilde{\psi}, \phi, h_1$ ($i = 1, 2$) have the Fourier representations:

$$X(\vec{x}, r) = \left(\frac{T}{V} \right)^{1/2} \sum_{\substack{\mathbf{q} \\ 0 < |\mathbf{k}| < \Lambda}} e^{i(\mathbf{k} \cdot \vec{x} - \omega_{\ell} r)} X(\mathbf{q}), \quad (X = \tilde{\psi}, \phi), \quad (8)$$

$$h_1(\vec{x}) = V^{-1/2} \sum_{0 < |\mathbf{k}| < \Lambda} e^{i\mathbf{k} \cdot \vec{x}} h_1(\mathbf{k}).$$

Now, we are in a position to calculate the two-point propagator

$G(\vec{x}, r; \{h\}) = \langle \psi(\vec{x}, r) \psi^*(0, 0) \rangle$ for a given RF configuration $\{h\}$. From (5) it immediately follows the factorization:

$$G(\vec{x}, r; \{h\}) = G_{\tilde{\psi}}(\vec{x}, r; \{h_1\}) \cdot G_{\phi}(\vec{x}, r; \{h_2\}), \quad (9)$$

where

$$G_{\tilde{\psi}}(\vec{x}, r; \{h_1\}) = \langle (\psi_0 + \tilde{\psi}(\vec{x}, r)) (\psi_0 + \tilde{\psi}(0, 0)) \rangle_{\mathcal{H}\{\tilde{\psi}, h_1\}}, \quad (10)$$

$$G_{\phi}(\vec{x}, r; \{h_2\}) = \langle e^{i[\phi(\vec{x}, r) - \phi(0, 0)]} \rangle_{\mathcal{H}\{\phi, h_2\}}, \quad (11)$$

and $\langle \dots \rangle_{\mathcal{H}\{\mathbf{x}, h_1\}}$ indicate the ensemble average with respect to $\mathcal{H}\{\mathbf{x}, h_1\}$. As we see, the propagators (10) and (11) depend only on the random configurations $\{h_1\}$ and $\{h_2\}$ of the real and imaginary parts of the RF respectively. Therefore, since it results in $\mathcal{P}\{h\} = \prod_{i=1}^2 \mathcal{P}\{h_i\}$, from (4) it immediately follows:

$$G(\vec{x}, r) = G_{\tilde{\psi}}(\vec{x}, r) \cdot G_{\phi}(\vec{x}, r), \quad (12)$$

$$G_{\tilde{\psi}}(\vec{x}, r) = [G_{\tilde{\psi}}(\vec{x}, r; \{h_1\})]_{\text{av}}^{(h_1)}, \quad (13)$$

$$G_{\phi}(\vec{x}, r) = [G_{\phi}(\vec{x}, r; \{h_2\})]_{\text{av}}^{(h_2)}. \quad (14)$$

In (13) and (14), $[\dots]_{\text{av}}^{(h_1)}$ indicates a quenched average with

respect to the Gaussian distribution of h_1 characterized by the random correlation functions:

$$[h_1(\mathbf{k}) h_1(\mathbf{k}')]_{\text{av}}^{(h_1)} = \frac{1}{2} \delta_{ij} \delta_{\mathbf{k}, -\mathbf{k}'} g(\mathbf{k}). \quad (15)$$

Thus, for a complete investigation of the role played by thermal, quantum and correlated RF fluctuations, it is sufficient to calculate explicitly $G_{\tilde{\psi}}(\vec{x}, r)$ and $G_{\phi}(\vec{x}, r)$ in the limit as $|\vec{x}| \rightarrow \infty$.

For $G_{\tilde{\psi}}(\vec{x}, r)$, from (10) and (13) with the quenched averages (15), it is easy to show that:

$$G_{\tilde{\psi}}(\vec{x}, r) = \psi_0^2 + \frac{T}{2V} \sum_{\mathbf{q}} \frac{e^{i(\mathbf{k} \cdot \vec{x} - \omega_{\ell} r)}}{-2r + c\mathbf{k}^{\sigma} + \omega_{\ell}^2} \left[1 + \frac{T^{-1} \delta \omega_{\ell, 0} g(\mathbf{k})}{4(-2r + c\mathbf{k}^{\sigma} + \omega_{\ell}^2)} \right], \quad (16)$$

where use is made of the Fourier transformation of $\mathcal{H}\{\tilde{\psi}, h_1\}$. Simple considerations analogous to those made for pure systems^{20, 27/} indicate that for dimensionalities of physical interest at arbitrary temperatures $G_{\tilde{\psi}}(\vec{x}, r) \rightarrow \psi_0^2$ for large distances. As concerns the phase part (14) of the two-point propagator $G(\vec{x}, r)$, for a given configuration of the RF imaginary part $\{h_2\}$, we can firstly write:

$$\exp\{-\mathcal{H}\{\phi, h_2\} - i[\phi(0, 0) - \phi(\vec{x}, r)]\} = \quad (17)$$

$$= \exp\left\{-\sum_{\mathbf{q}} [\psi_0^2 (c\mathbf{k}^{\sigma} + \omega_{\ell}^2) \cdot |\phi(\mathbf{q})|^2 + \gamma(\mathbf{q}) \phi(\mathbf{q})]\right\},$$

where $0 < |\mathbf{k}| < \Lambda$

$$\gamma(\mathbf{q}) = -\psi_0 T^{-1/2} \delta \omega_{\ell, 0} h_2(-\mathbf{k}) + i \left(\frac{T}{V} \right)^{1/2} (1 - e^{i(\mathbf{k} \cdot \vec{x} - \omega_{\ell} r)}). \quad (18)$$

Then, after simple Gaussian integrations, we have

$$G_{\phi}(\vec{x}, r; \{h_2\}) = \exp\left\{-\frac{1}{2\psi_0^2 (c\mathbf{k}^{\sigma} + \omega_{\ell}^2)} \left(\frac{T}{V} (1 - \cos(\mathbf{k} \cdot \vec{x} - \omega_{\ell} r)) + i \frac{\psi_0}{V^{1/2}} \delta \omega_{\ell, 0} \left[a(\mathbf{k}) (1 - \cos(\mathbf{k} \cdot \vec{x} - \omega_{\ell} r)) + b(\mathbf{k}) \sin(\mathbf{k} \cdot \vec{x} - \omega_{\ell} r) \right] \right)\right\}, \quad (19)$$

where $a(\mathbf{k})$ and $b(\mathbf{k})$ are the real and imaginary parts of $h_2(\mathbf{k})$. Thus, making use of the quenched averages (15) we find:

$$G_{\phi}(\vec{x}, r) = \exp\{-\lambda^{(P)}(\vec{x}, r) - \lambda^{(R)}(\vec{x})\} = G_{\phi}^{(P)}(\vec{x}, r) G_{\phi}^{(R)}(\vec{x}), \quad (20)$$

where:

$$\lambda^{(P)}(\vec{x}, r) = \frac{T}{V} \sum_{\substack{q \\ 0 < |\vec{k}| < \Lambda}} \frac{1 - \cos(\vec{k}\vec{x} - \omega_{\ell} r)}{2\psi_0^2 (c\vec{k}^\sigma + \omega_{\ell}^2)}, \quad (21)$$

$$\lambda^{(R)}(\vec{x}) = \frac{1}{V} \sum_{\substack{\theta \\ 0 < |\vec{k}| < \Lambda}} \frac{\Delta_1 + \Delta_2 k^\theta}{8c^2 \psi_0^2 k^{2\sigma}} (1 - \cos \vec{k}\vec{x}). \quad (22)$$

Now, for our purposes, we must investigate the asymptotical behaviour of $\lambda^{(P)}(\vec{x}, r)$ and $\lambda^{(R)}(\vec{x})$ for large $|\vec{x}|$ in the thermodynamic limit for $V \rightarrow \infty$ for several values of d, σ, θ and Δ_1 ($1 = 1, 2$). Firstly, by making the sum over frequencies in (21), for the pure part of the phase propagator we obtain:

$$\lambda^{(P)}(\vec{x}, r) = \frac{1}{4\psi_0^2} \frac{1}{V} \sum_{\substack{q \\ 0 < |\vec{k}| < \Lambda}} \frac{1}{(c\vec{k}^\sigma)^{1/2}} \left\{ \left[2n\left(\frac{(c\vec{k}^\sigma)^{1/2}}{T}\right) + 1 \right] + \cos \vec{k}\vec{x} \left[e^{r(c\vec{k}^\sigma)^{1/2}} n\left(\frac{(c\vec{k}^\sigma)^{1/2}}{T}\right) + e^{-r(c\vec{k}^\sigma)^{1/2}} \left[n\left(\frac{(c\vec{k}^\sigma)^{1/2}}{T}\right) + 1 \right] \right] \right\}, \quad (23)$$

where $n(x) = [e^x - 1]^{-1}$ is the Bose function. Then, in the "classical limit" for $(c\Lambda^\sigma)^{1/2} / T \ll 1$, at $T \neq 0$, we find for $|\vec{x}| \rightarrow \infty$:

$$\lambda^{(P)}(\vec{x}, r) \sim \begin{cases} B(d, \sigma) \frac{T}{c\psi_0^2} |\vec{x}|^{\sigma-d}, & d < \sigma, \\ K_\sigma \frac{T}{c\psi_0^2} \ln(\Lambda |\vec{x}|), & d = \sigma, \\ \text{a finite value,} & d > \sigma, \end{cases} \quad (24)$$

where

$$B(d, \sigma) = \frac{2^{2-d} \pi^{-(d+1)/2}}{\Gamma(\frac{1}{2}(d-1))} \int_0^\infty dx \cdot x^{d-\sigma-1} \int_0^{\pi/2} d\phi \cdot \sin^{d-2} \phi \sin^2\left(\frac{x}{2} \cos \phi\right).$$

In the "quantum regime" $T=0$, from (23) it follows:

$$\lambda^{(P)}(\vec{x}, r) \sim \begin{cases} +\infty, & d \leq \sigma/2, \\ \frac{K_d \Lambda^{d-\sigma/2}}{4c^{1/2} \psi_0^2 (d-\sigma/2)}, & d > \frac{\sigma}{2}. \end{cases} \quad (25)$$

In (24)-(25) $K_d = \pi^{-d/2} 2^{1-d} / \Gamma(d/2)$. The corresponding asymptotical expressions $G_\phi^{(P)}(\vec{x}, r)$ immediately follow from (24)-(25). Of course, in the classical regime, the results for classical systems with long-range interactions are reproduced (see ref. /20/ for $\sigma = 2$). As concerns the random part $G_\phi^{(R)}(\vec{x})$ of $G_\phi(\vec{x}, r)$ notice that the thermal and quantum fluctuations do not play any role and therefore the results related to the RF effects will be true for arbitrary temperature. For obtaining its asymptotical behaviour for $|\vec{x}| \rightarrow \infty$, it is convenient to rewrite $\lambda^{(R)}(\vec{x})$ as:

$$\lambda^{(R)}(\vec{x}) = \frac{1}{8c^2 \psi_0^2} \left[\Delta_1 I_{2\sigma}^{(d)}(\vec{x}) + \Delta_2 I_{2\sigma-\theta}^{(d)}(\vec{x}) \right], \quad (26)$$

where

$$I_\mu^{(d)}(\vec{x}) = \int_{\substack{q \\ 0 < |\vec{k}| < \Lambda}} \frac{d^d k}{(2\pi)^d} \frac{1 - \cos \vec{k}\vec{x}}{k^\mu} \quad (\mu = 2\sigma, 2\sigma - \theta).$$

For $\Lambda |\vec{x}| \gg 1$ we find:

$$I_\mu^{(d)}(\vec{x}) \approx \begin{cases} A(d, \mu) |\vec{x}|^{\mu-d}, & d < \mu, \\ 2K_\mu \ln(\Lambda |\vec{x}|), & d = \mu, \\ \text{a finite value,} & d > \mu. \end{cases} \quad (27)$$

where

$$A(d, \mu) = \frac{\pi^{1/2} \Gamma(d/2) K_d}{\Gamma(\mu/2)} \frac{\Gamma(\frac{1}{2}(1-d+\mu))}{\Gamma(1-d+\mu)} \frac{\cos(\frac{\pi}{2}(\mu-d))}{\sin(\pi(\mu-d))} > 0.$$

In order to have a clear physical picture of the competition of thermal, quantum and random fluctuations and the correct asymptotical behaviour of the physical propagator $G(\vec{x}, \tau)$ for large distances both in the classical and quantum regimes, we now collect together the results (24)-(27) separately in the cases: i) $\Delta_1 \neq 0, \Delta_2 = 0$, ii) $\Delta_1 = 0, \Delta_2 \neq 0$, arbitrary θ ; iii) $\Delta_1 \neq 0, \Delta_2 \neq 0$, arbitrary θ . Of course i), ii) for $\theta = 0$ and iii) for $\theta \geq 0$ correspond to short-range correlated RFs.

i) $\Delta_1 = \Delta \neq 0, \Delta_2 = 0.$

From (26)-(27) one has for $|\vec{x}| \rightarrow \infty$:

$$\lambda^{(R)}(\vec{x}) \approx \frac{\Delta}{8c^2 \psi_0^2} \begin{cases} A(d, 2\sigma) |\vec{x}|^{2\sigma-\theta}, & d < 2\sigma, \\ 2K_{2\sigma} \ln(A |\vec{x}|), & d = 2\sigma, \\ \text{a finite value,} & d > 2\sigma. \end{cases} \quad (28)$$

Then, since $\lambda^{(P)}(\vec{x}, r) \rightarrow$ a finite value for $d > \sigma$ at $T \neq 0$ and $d > \sigma/2$ at $T=0$, we have both in the classical and quantum regimes:

$$G(\vec{x}, r) - G^{(R)}(\vec{x}) = \begin{cases} \exp\left\{-\frac{\Delta}{8c^2 \psi_0^2} A(d, 2\sigma) |\vec{x}|^{2\sigma-d}\right\}, & d < 2\sigma, \\ \exp\left\{-\frac{\Delta}{4c^2 \psi_0^2} K_{2\sigma} \ln(A |\vec{x}|)\right\}, & d = 2\sigma, \\ \text{a constant value,} & d > 2\sigma, \end{cases} \quad (29)$$

as $|\vec{x}| \rightarrow \infty$. Thus, for $d < 2\sigma$, $G(\vec{x}, r) \rightarrow 0$ for large distances. Of course, this is "inconsistent" with original assumption of the existence of an ordered phase and allows us to conclude that a long-range order is impossible for such dimensionalities but not for $d > 2\sigma$. This implies $d_{CL} = 2\sigma$ without ambiguity for $d = d_{CL}$ as usual it happens when arguments are used for RF classical systems^{/10/}. In conclusion, when short-range correlated RFs are present, thermal and quantum fluctuations are irrelevant with respect to the RF ones and these are sufficiently strong to destroy long-range order in the region of dimensionalities where the pure systems show an ordered phase.

ii) $\Delta_1 = 0, \Delta_2 \neq 0, \text{arbitrary } \theta.$

For $|\vec{x}| \rightarrow \infty$ eq. (26) becomes:

$$\lambda^{(R)}(\vec{x}) \approx \frac{\Delta_2}{8c^2 \psi_0^2} \begin{cases} A(d, 2\sigma - \theta) |\vec{x}|^{2\sigma - \theta - d}, & d < 2\sigma - \theta, \\ 2K_{2\sigma - \theta} \ln(A |\vec{x}|), & d = 2\sigma - \theta, \\ \text{a finite value,} & d > 2\sigma - \theta. \end{cases} \quad (30)$$

In the present case, it is necessary to distinguish the two cases $2\sigma - \theta > d_{CL}^{(P)}$ and $2\sigma - \theta < d_{CL}^{(P)}$, where $d_{CL}^{(P)} = \sigma$ in the classical regime and $d_{CL}^{(P)} = \sigma/2$ in the quantum regime for pure systems. Since $\lambda^{(P)}(\vec{x}, r) \rightarrow$ a finite value for $d > d_{CL}^{(P)}$ both at $T \neq 0$ and $T=0$, for $2\sigma - \theta > d_{CL}^{(P)}$ it follows that:

$$G(\vec{x}, r) \sim \begin{cases} \exp\left\{-\frac{\Delta_2}{8c^2 \psi_0^2} A(d, 2\sigma - \theta) |\vec{x}|^{2\sigma - \theta - d}\right\}, & d < 2\sigma - \theta, \\ \exp\left\{-\frac{\Delta_2}{4c^2 \psi_0^2} K_{2\sigma - \theta} \ln(A |\vec{x}|)\right\}, & d = 2\sigma - \theta, \\ \text{a finite quantity,} & d > 2\sigma - \theta. \end{cases}$$

Thus, the thermal and quantum fluctuations are irrelevant again and the RF ones destroy any long-range order for $d \leq 2\sigma - \theta$. Therefore, in the present case, $d_{CL} = 2\sigma - \theta$ for arbitrary T . The situation appears different for $2\sigma - \theta < d_{CL}^{(P)}$ since $\lambda^{(R)}(\vec{x}) \rightarrow$ a finite value and $\lambda^{(P)}(\vec{x}, r) \rightarrow \infty$ for large distances when $2\sigma - \theta < d_{CL}^{(P)}$. The random fluctuations are now irrelevant with respect to the thermal and quantum ones and it results

$$d_{CL} = d_{CL}^{(P)} = \begin{cases} \sigma, & T \neq 0, \\ \sigma/2, & T = 0, \end{cases}$$

as in the pure quantum systems. Of course, for $\theta = 0$ the short-range case (i) is reproduced with $\Delta = \Delta_2$.

iii) $\Delta_1 \neq 0, \Delta_2 \neq 0, \text{arbitrary } \theta.$

For $\theta \geq 0$, it is immediate to see that, as expected, the short-range random case (i) is reproduced ($d_{CL} = 2\sigma$) with $\Delta = \Delta_1$ if $\theta > 0$ and $\Delta = \Delta_1 + \Delta_2$ if $\theta = 0$. If $\theta < 0$, we have that it is always $2\sigma - \theta > 2\sigma$ and in (26) the term in Δ_2 dominates for large distances and the results (30) and (31) are formally true. Then, one finds $d_{CL} = 2\sigma + |\theta|$ for arbitrary T since both thermal and quantum fluctuations appear irrelevant with respect to the random ones.

The previous analyses indicate that the relevance of the RF fluctuations on the destruction of long-range order in classical and quantum systems may depend sensibly on appropriate combinations of the parameters σ and θ measuring the power of the fall-off of the involved interactions and disorder correlation functions. Such a point of view and the possibility of having long-range order at physical dimensionalities may have great relevance for a correct interpretation of the available experimental results and for identifying the type of defects involved in the laboratory samples also when the quantum regime is approached.

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Бусиелло Г., Де Чезаре Л., Рабуффо И.

E17-85-926

Отсутствие дальнего порядка в квантовых сегнетоэлектриках с коррелированными случайными полями

Метод, эффективный при трактовке разрушения дальнего порядка в чистых классических системах с непрерывной симметрией, распространяется на класс квантовых систем с коррелированными случайными полями. Дана обоснованная оценка нижней критической размерности таких систем в терминах параметров, характеризующих радиус учитываемых взаимодействий и случайных корреляций. Эта оценка следует из рассмотрения конкуренции между термическими, квантовыми и случайными полевыми флуктуациями.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1985

Busiello G., De Cesare L., Rabuffo I.

E17-85-926

Absence of Long-Range Order in Quantum Ferroelectrics with Correlated Random Fields

A physically reasonable method for treating the destruction of long-range order in pure classical systems with continuous symmetry is extended to include a class of quantum systems in the presence of correlated random fields. Then, a reliable estimate of the lower critical dimensionality for such random systems is given in terms of the parameters characteristic of the range of the involved interactions and random correlations. This has been realized by means of an appropriate treatment of the competition among thermal, quantum and random field fluctuations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1985