ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
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ИССЛЕДОВАНИЙ

Дубна

E-2247

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ДАН СССР, 1966, т167, №4, с.792-794

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1965

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Submitted to DAN

It was emphasized in [1,2] that the S-matrix invariance under the group $SL(6)$ [3-9] (or under the group $\tilde{U}(12)$ [10,11]) is inconsistent with the unitarity condition. However, since the ware equations even for free particles break down the symmetry $SL(6)$ there are no reasons to require the S-matrix invariance under the group $SL(6)$. We will show that if the breakdown of the symmetry $SL(6)$ is taken into account then the S-matrix is unitary. Moreover, the unitarity condition itself is invariant under the group $SL(6)$ if instead of the usual 4-momenta there are 36-dimensional momenta.

As has been in a number of papers, the invariance under the group $SL(6)$ requires an existence of the 36-dimensional momenta $(P)^{A}_{\dot B}$ and $(P)^{\dot A}_{B}$, $A=(\alpha a)$, $\alpha=1,2,3$, $a=1,2$, which transform as corresponding spinors of the group $SL(6)$. The wave function for the quarks invariant under the group $SL(6)$ has the form

$$(iP)^{\dot A}_{B}\ \phi^{B} + m\chi^{\dot A} = 0, \qquad (iP)^{A}_{\dot B}\ \chi^{\dot B} + m\phi^{A} = 0. \tag{1}$$

If in this equation we made a substitution

$$(P)^{\dot A}_{B} \to (p)^{\dot a}_{b}\ \delta^{\alpha}_{\beta}, \qquad (P)^{A}_{\dot B} \to (p)^{a}_{\dot b}\ \delta^{\alpha}_{\beta}, \tag{2}$$

then we would obtain the Dirac equation. The matrix elements of the scattering processes and vertex functions contain explicitly not only the wave functions of the wave initial and final particles but their momenta as well[1)]. If instead of the 4-momenta the 36-dimensional momenta are introduced then the matrix elements will be invariant under the group $SL(6)$, and after the substitution (2) the symmetry $SL(6)$ is broken.

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1) The terms which contain the momenta explicitly are called irregular The existence of such irregular structures in the matrix elements was first pointed out in [7] by one of the authors where the so-called spurion formalism of the broken symmetry $SL(6)$ was suggested. Then this was shown independently in paper [12]. In paper [8] devoted to a study of the structure of vector and axial currents the contributions from all irregular structures are taken into account.

As an example we consider the scattering of neutral scalar meson (singlet) on the quarks. According to the method suggested in ref. /7/ we consider first the 4-momenta of the mesons and the quarks the initial and the final states k_1, p_1, k_2, p_2, respectively as the components of the tensors $(K_1)^{A}_{\dot{B}}$, $(P_1)^{A}_{\dot{B}}$ and $(K_1)^{\dot{A}}_{B}$, $(P_1)^{\dot{A}}_{B}$. A pair of the tensors K_1 is equivalent to a system consisting of nine 4-vectors $(k_1)^j_\mu$ and of nine 4-pseudovectors $(\ell_1)^j_\mu$; $j=0,1,\dots,8$; $\mu=1,\dots,4$; while a pair of tensors P_1 – to the system of 4-vectors $(p_1)^j_\mu$ and 4-pseudovectors $(q_1)^j_\mu$.

$$
\begin{aligned}
(K_1)^{A}_{\dot{B}} &= \frac{1}{\sqrt{2}} \sum_{\mu=1}^{4} \sum_{j=0}^{8} (\lambda_j)^{a}_{\beta} (\sigma_\mu)^{\dot{a}}_{b} \left[(k_1)^j_\mu + (\ell_1)^j_\mu \right], \\
(K_1)^{\dot{A}}_{B} &= \frac{1}{\sqrt{2}} \sum_{\mu=1}^{4} \sum_{j=0}^{8} (\lambda_j)^{a}_{\beta} (\sigma_\mu)^{\dot{a}}_{b} \left[(k_1)^j_\mu - (\ell_1)^j_\mu \right],
\end{aligned} \tag{3}
$$

and similarly for the tensors P_1. The invariant square of the momentum K is, for instance, as follows

$$
\frac{1}{3} (K)^{A}_{\dot{B}} (K)^{\dot{B}}_{A} = \sum_{j=0}^{8} \left[(k^j)^2 - (\ell^j)^2 \right] = \sum_{j,\mu} \left[k^j_\mu k^j_\mu - \ell^j_\mu \ell^j_\mu \right]. \tag{4}
$$

The invariant element of the volume is equal to

$$
\frac{1}{(2)^{36}} \prod_{A,\dot{B}} d\,(k)^{A}_{\dot{B}} \prod_{\dot{C},D} d\,(k)^{\dot{C}}_{D} = \prod_{j=0}^{8} d^4 k^j \, d^4 \ell^j , \tag{5}
$$

and the invariant δ-function is of the form

$$
(2)^{36} \, \delta^{36} (K^{A}_{\dot{B}}) \, \delta^{36} (K^{\dot{A}}_{B}) = \delta^{36} (k^j) \, \delta^{36} (\ell^j). \tag{6}
$$

The matrix element of the process under consideration which contains the 36-momenta K_1 and P_1 which is invariant under the group $SL(6)$ can be written in the following manner

$$
\begin{aligned}
M_{fi} = (2\pi)^{72} \, & \delta^{36} (p^j_1 + k^j_1 - p^j_2 - k^j_2) \, \delta^{36} (q^j_1 + \ell^j_1 - q^j_2 - \ell^j_2) \{ \\
& A(P_2, K_2; P_1, K_1) [\bar{\phi}(P_2)_{\dot{A}} \, \chi(P_1)^{\dot{A}} + \bar{\chi}(P_2)_A \, \phi(P_1)^A] + B(P_2, K_2; P_1, K_1)
\end{aligned} \tag{7}
$$

$$[\bar{\phi}(P_2)_{\dot{A}}(i\frac{K_1+K_2}{2m})^{\dot{A}}_{B}\varphi(P_1)^{B}+\bar{\chi}(P_2)_{A}(i\frac{K_1+K_2}{2m})^{A}_{\dot{B}}\chi(P_1)^{\dot{B}}]\}$$

For the meson with mass μ, e.g., the phase volume invariant under the group SL(6) is equal to

$$\frac{1}{(2\pi)^{72}}\prod_{j=0}^{8} d^4k^j\, d^4\ell^j\, 2\pi\,\delta\,(\sum_j (k^j)^2-\sum_j(\ell^j)^2+\mu^2)\,. \tag{8}$$

The unitarity condition in the two-particle approximation is

$$\operatorname{Im} A(P_2,K_2;P_1,K_1)\,[\phi(P_2)_{\dot{A}}\chi(P_1)^{\dot{A}}+\bar{\chi}(P_2)_A\phi(P_1)^{A}]+$$

$$\operatorname{Im} B(P_2,K_2;P_1,K_1)\,[\bar{\varphi}(P_2)_{\dot{A}}(i\frac{K_1+K_2}{2m})^{\dot{A}}_{B}\phi(P_1)^{B}+\ldots\ldots]=$$

$$\frac{m}{(2\pi)^{72}}\int d^4p^j\, d^4q^j\, d^4k^j\, d^4\ell^j\,\delta^{36}(p^j+k^j-p_1^j-k_1^j)\,\delta^{36}(q^j+\ell^j-q_1^j-\ell_1^j)$$

$$(2\pi)^2\,\delta\,(\Sigma(p^j)^2-\Sigma(q^j)^2+m^2)\,\delta\,(\Sigma(k^j)^2-\Sigma(\ell^j)^2+\mu^2)\sum_r\{A^*(P,K;P_2,K_2) \tag{9}$$

$$[\bar{\phi}(P_2)_{\dot{A}}\chi^r(P)^{\dot{A}}+\bar{\chi}(P_2)_A\phi^r(P)^{A}]+B^*(P,K;P_2,K_2)[\ldots\ldots]\}\,.$$

$$\{A(P,K;P_1,K_1)[\bar{\varphi}^r(P)_{\dot{B}}\chi(P_1)^{\dot{B}}+\bar{\chi}^r(P)_B\varphi(P_1)^{B}]+$$

$$B(P,K;P_1,K_1)[\ldots\ldots\ldots]\}\,.$$

For the summation over the polarization of the intermediate states we use the formula

$$\sum_r \phi^r_{\dot{A}}\,\varphi^{rB}=\tfrac{1}{2}\,(\frac{-iP}{m})^{B}_{\dot{A}}\,,\qquad \sum_r \chi^r_A\,\chi^{r\dot{B}}=\tfrac{1}{2}\,(\frac{-iP}{m})^{\dot{B}}_{A}\,,$$

$$\sum_r \phi^r_{\dot{A}}\,\chi^{r\dot{B}}=\tfrac{1}{2}\,\delta^{\dot{B}}_{\dot{A}}\,,\qquad \sum_r \chi^r_A\,\phi^{rB}=\tfrac{1}{2}\,\delta^{B}_{A}\,. \tag{10}$$

It is evident that the unitarity condition (9) is invariant under the group SL(6).

The physical scattering amplitude is obtained from the invariant amplitude by the substitution

$$A(P_2, K_2; P_1, K_1) \to \prod_{j=1}^{8} (2\pi)^4 \delta^4(p_1^j - p_2^j) \prod_{j=0}^{8} (2\pi)^4 \delta^4(q_1^j - q_2^j) \tag{11}$$

$$\theta [(p_1)_0^0] A(s,t),$$

and similarly for B . At the same time for the momenta of initial particles we make a substitution (2), i.e. we keep only the physical 4-momenta $(p_i)^\rho_\mu$ and $(k_i)^\rho_\mu$. Then the unitarity condition (9) together with the formula (10) gives the usual unitarity condition

$$\mathrm{Im}\, A(s,t) + \mathrm{Im}\, B(s,t)\, i \frac{\hat{k}_1 + k_2}{2m} = \frac{1}{(2\pi)^2} \int d^4p\, d^4k\, \theta(p^0)\theta(k^0)$$

$$\delta(p^2 + m^2)\, \delta(k^2 + \mu^2) [A^*(s,t'') + B^*(s,t'')\, i \frac{\hat{k}_2 + \hat{k}}{2m}] \frac{m - i\hat{p}}{2} \tag{12}$$

$$[A(s,t') + B(s,t')\, i \frac{\hat{k} + \hat{k}_1}{2m}].$$

Thus, if instead of the usual 4-momenta use is made of the 36-dimensional momenta, then the theory is completely invariant under the group SL(6) : the wave equation, S-matrix and the unitarity condition are invariant. In passing to the physical amplitudes the symmetry is violated: the wave equations, S-matrices and the unitarity condition are SL(6) non-invariant. The authors of a number of papers require a strict invariance for the physical amplitude what in the considered case leads to the condition

$$B(s,t) \equiv 0.$$

It is evident that this incorrect requirement leads to a contradiction with the unitarity condition and there is no problem about the inconsistency of the group SL(6) with the unitarity condition.

We point out in conclusion that for the given scattering process of a singlet meson on the quark the broken symmetry SL(6) gives nothing new if compared with the unitary symmetry. However, for higher representations the broken symmetry SL(6) yields new consequences. So, for instance in the unitary symmetry the matrix elements of the vector and axial currets for the octet depend on 12 form-factors, while in the broken symmetry SL(6) these

12 form-factors are expressed in terms of 8 independent form-factors[2)], if all the irregular structures are taken into account[8]. Moreover, the form-factors for the decuplet and the form-factors of the transitions between the octet and the decuplet are also expressed in terms of these 8 independent form-factors.

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Received by Publishing Department
on June 29, 1965.

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2) In papers[6,9] the non-regular structures are not taken into account. Therefore, all the form-factors are expressed in terms of 5 independent functions.