ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
Дубна

E- 2156

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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ЯФ, 1965, т.2, в.4, с.762-767.

1965

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1. A great number of papers have appeared recently in which SU(6) - symmetry was applied to the elementary particle physics /1-9/. The predictions of SU(6) - invariance which concern the "static" properties of particles (particle classification by multiplets, mass formulae, the ratio of the magnetic moments of the neutron and the proton, the F/D ratio for the axial current etc) are in excellent agreement with the experimental data.

In applying SU(6) - symmetry to the problems of particle collisions one is met with a difficulty connected with a nonrelativistic character of the group SU(6). On the other hand, besides SU(6) invariant interactions, those breaking SU(6) are also essential.

Johnson and Treiman /10/ applied the exact SU(6) - symmetry to the meson-baryon forward scattering amplitude and obtained, with the help of the optical theorem, some relations between the total cross sections for π and K mesons interaction with nucleons. These relations are in good agreement /10-11/ with experiment in a wide energy range.

In this note we will treat baryon-baryon scattering. Since in baryon-baryon interactions the essential role belongs to tensor and spin-orbital forces breaking SU(6) symmetry and leading, in particular, in nucleon-nucleon interactions to considerable polarization effects, it is reasonable, in applying SU(6) symmetry (or its subgroup SU(4)), to restrict ourselves to a consideration of collisions at not too high energies in the S-state where these forces do not play any role x). On the other hand the well-known A_3^3 violations of the unitary symmetry SU(3) lead to an appreciable breakdown of SU(6) and its subgroup SU(2) × SU(3) what makes it very difficult to compare the predictions of SU(6) with experiment for the S-wave as well. However, the SU(4) subgroup of the group SU(6) may turn out to be broken down essentially weaker (at least the subgroup SU(2) × SU(2) of SU(4) is a symmetry good enough for the S-waves). Therefore, the main emphasis is laid upon the application of SU(4) - symmetry to nucleon-nucleon scattering. Only in the end we shall state briefly the results following from SU(6) - symmetry.

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x) Since the tensor forces lead to the ${}^3S_1 \rightleftarrows {}^3D_1$ transitions, the application of SU(6) or SU(4) - symmetry for the S-waves should be limited to the energy region where the cosine of the mixing parameter ϵ_1 is not very much different from unity.

II. Since in the SU(6)- symmetry the octet and the decuplet of baryons form a 56-plet, then in the framework of SU(4) the nucleons and (3,3)-resonances should be united[4] into a multiplet of dimension 20 which is described by a completely symmetric tensor of rank three in the complex four-dimensional space x). This tensor is reducible under the direct product of the spin and isotopic groups SU(2) × SU(2) and can be written down as

$$\psi^{ABC} \equiv \psi^{\alpha i,\beta j,\gamma k} = d^{\alpha\beta\gamma,ijk} + \frac{1}{3\sqrt{2}}\left[\epsilon^{\alpha\beta}\left(2\epsilon^{ij}\chi^{\gamma,k} + \epsilon^{jk}\chi^{\gamma,i}\right) + \epsilon^{\beta\gamma}\left(\epsilon^{ij}\chi^{\alpha,k} + 2\epsilon^{jk}\chi^{\alpha,i}\right)\right] \tag{1}$$

Here α, β, γ and i, j, k are the isotopic and spin indices, and $\epsilon^{\alpha\beta}$ and ϵ^{ij} are the Levi - Civita symbols in two dimensions. The tensor $d^{\alpha\beta\gamma,ijk}$ is symmetric with respect to any permutation of the isotopic (and spin) indices and describes the (3,3) resonance, $\chi^{\alpha,i}$ stands for the nucleon state.

The direct product of representations 20^* and 20 splits into four irreducible representations

$$20^* \times 20 = 1 + 15 + 84 + 300 \tag{2}$$

The baryon-baryon scattering amplitude is determined, therefore, by four scalars and can be written in the following form

$$\begin{aligned} M = {} & a(\theta)\,\bar{\Psi}_{ABC}(f1)\,\Psi^{ABC}(i1)\,\bar{\Psi}_{LMN}(f2)\,\Psi^{LMN}(i2) + \\ & + b(\theta)\,\bar{\Psi}_{ABC}(f1)\,\Psi^{ABN}(i1)\,\bar{\Psi}_{LMN}(f2)\,\Psi^{LMC}(i2) + \\ & + c(\theta)\,\bar{\Psi}_{ABC}(f1)\,\Psi^{AMN}(i1)\,\bar{\Psi}_{LMN}(f2)\,\Psi^{LBC}(i2) + \\ & + d(\theta)\,\bar{\Psi}_{ABC}(f1)\,\Psi^{LMN}(i1)\,\bar{\Psi}_{LMN}(f2)\,\Psi^{ABC}(i2) \, . \end{aligned} \tag{3}$$

In this expression $\Psi^{ABC}(i1)$ and $\Psi^{ABC}(f1)$ are tensors describing the initial and the final state of the first baryon; $\Psi^{MNL}(i2)$ and $\Psi^{MNL}(f2)$ have a similar meaning for the second baryon, θ is the angle between the initial and the final momenta of the first baryon.

x) This is the difference from Wigner's theory of supermultiplets[12] in which the nucleons are transformed under the 4 - dimensional representation of the group SU(4)

The generalized Pauli principle leads to the following relations between the four amplitudes entering (3)

$$a(\pi-\theta)=-d(\theta)$$
$$b(\pi-\theta)=-c(\theta) \qquad (4)$$

Singling out from (3), with the help of (1), nucleon-nucleon scattering and going over to the matrix form we get the following expression for the matrix of N-N collisions[x)]

$$M_{NN}=A(\theta)+B(\theta)(P_\sigma+P_\tau)+D(\theta)\,P_\sigma P_\tau \qquad (5)$$

where

$$P_\sigma=\tfrac{1}{2}(1+\vec{\sigma}_1\vec{\sigma}_2) \qquad\qquad P_\tau=\tfrac{1}{2}(1+\vec{\tau}_1\vec{\tau}_2)$$

are the spin and isotopic spin exchange operators, $\vec{\sigma}_i$ and $\vec{\tau}_i$ are the spin and isotopic spin operators of the i-th nucleon, while $A(\theta)$, $B(\theta)$, and $D(\theta)$ are connected with the amplitudes introduced earlier by the following relations

$$A(\theta)=a(\theta)+\frac{22}{81}b(\theta)+\frac{25}{81}c(\theta)=-D(\pi-\theta)$$

$$B(\theta)=-\frac{8}{81}[b(\theta)+c(\theta)]=-B(\pi-\theta) \qquad (6)$$

$$D(\theta)=d(\theta)+\frac{25}{81}b(\theta)+\frac{22}{81}c(\theta)=-A(\pi-\theta)$$

As it should be expected, SU(4)-invariance imposes rather severe limitations on a possible form of the matrix for nucleon-nucleon scattering. Out of ten functions of the scattering angle and energy which characterize the N-N scattering matrix in the general case, there remain only three in the framework of SU(4)-symmetry. The scattering matrix (5) means that the polarization of final particles is equal to zero when the initial nucleons are unpolarized. This also implies that in the depolarization tensor $D_{ik}\approx\delta_{ik}$ only one component is different from zero etc. There are, of course, no reasons to believe that (5) is an approximation good enough to describe nucleon-nucleon scattering. One

x) To the collision matrix (5) there corresponds the nucleon-nucleon potential $V_{NN}=V_1+V_2(P_\sigma+P_\tau)+V_3\,P_\sigma P_\tau$ which involves, besides Wigner's and Majorana's forces, those of Bartlet and Heisenberg. This potential is symmetric, however, with respect to the substitution of spins of both nucleons by their isotopic spins and vice versa $\vec{\sigma}_i \rightleftarrows \vec{\tau}_i$.

may, however, hope, as was mentioned above, that $SU(4)$ - symmetry is a reasonable approximation for the S-wave. In this case the amplitudes are independent of the scattering angle, and from the Pauli priciple it follows that

$$c = -b, \qquad d = -a \tag{7}$$

From (5) and (7) we get that the N-N scattering matrix in the S-state is equal to

$$M_{NN} = (a - \frac{b}{27})(1 - P_\sigma P_\tau) \tag{8}$$

From here follows the equality of n-p scattering amplitudes in the $^1S_0$ and $^3S_1$ states

$$M_{np}(^1S_0) = M_{np}(^3S_1) \tag{9}$$

When comparing this relation with the experimental data one should take a region of not too low energies. The case is that low energy S - scattering has a resonance character and is determined by the presence of the bound and virtual states in the n-p system. In the exact $SU(4)$ symmetry these levels must coincide. The symmetry-breaking interactions will lead to the splitting of these levels and due to the resonance character of the scattering they will affect strongly the values of the scattering lengths.

If we now turn to the results of the phase shift analysis of p-p and n-p scatterings in the best studied energy region (see Fig.1 and the phase shift curves in Kazarinov's report /13/), we see the following picture. The shapes of the curves showing the energy dependence of the phase shifts in different states differ strongly from one another. However, the curves for the energy dependence of the phase shifts in the $^1S_0$ and $^3S_1$ states are close in the energy region from 100 up to 400 MeV, in qualitative agreement with (9). At 310 MeV these phase shifts coincide with a good accuracy.

One may attempt a phase shift analysis, (9) being imposed as an additional condition, and see how well the experimental data will be fitted. Such an analysis was made by us at 147, 210 and 310 MeV. As initial data we used the phase sets given in /13/. It turned out that at energies 210 and 310 MeV new phase sets describe experimental data just as well as the old ones : the ratio $\frac{\chi^2}{\overline{\chi^2}}$ remains practically unchanged. At an energy of 147 MeV the situation is worse. To a new set there corresponds $\frac{\chi^2}{\overline{\chi^2}} =$ 1.8 instead of 0.9 for the old set.

To summarize, one may say that the values of the scattering phase shifts in the $^1S_0$ and $^3S_1$ - states in the region of not too high energies where there is a hope to fulfil relation (9) do not contradict this relation so as its validity could be given up.

If we believe, like Johnson and Treiman /10/ did in the case of SU(6)-symmetry, that the requirements of SU(4) - invariance may be applied to the forward scattering matrix (with account of all partial waves) then, using the optical theorem, it is easy to obtain relations between the total cross sections of n-p and p-p interactions in the case of collision of polarized particles[x]. Since the forward scattering matrix is determined by three independent amplitudes then there are obviously no relations between the total cross sections of n-p and p-p interactions with unpolarized particles. A general expression for the total cross section for the interaction of polarized particles ($\vec{P}_1$ and $\vec{P}_2$ are the polarizations of the beam and the target, $\vec{k}$ is the unit vector in a direction of the incident beam momentum) has the form

$$\sigma = \sigma_0 + \sigma_1 \vec{P}_1 \cdot \vec{P}_2 + \sigma_2 \vec{P}_1 \cdot \vec{k} \; \vec{P}_2 \cdot \vec{k} \qquad (10)$$

Applying the optical theorem connecting the coefficients of the forward scattering matrix (5) with the cross sections σ_i /14, 15/ we get

$$\sigma_0 (pp) - \sigma_0 (np) = \sigma_1 (pp) + \sigma_1 (np) \qquad (11.1)$$

$$\sigma_2 (np) = \sigma_2 (pp) = 0 \qquad (11.2)$$

The equality (11.2) is not crucial with respect to SU(4) - symmetry since it holds in the case when the forward scattering matrix is invariant only under the transformations of the group SU(2) × SU(2). The relation (11.1) can be rewritten in the form

$$\sigma_t (np) = ½ (\sigma_s (pp) + \sigma_t (pp)) \qquad (12)$$

where σ_s and σ_t are the total interaction cross sections in the singlet and the triplet states. These relations can be verified in experiments with a polarized beam and a polarizet target.

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x) Although the matrix (5) does not lead to the polarization of final particles by collision of unpolarized nucleons it gives a definite correlation of polarizations what is reflected in the total interaction cross section of polarized particles.

III. Consider now baryon-baryon scattering in $SU(6)$ - symmetry. The procedure of obtaining the relations resembles in its essentials that given above for the case of $SU(4)$. The only difference is that the baryons are described now by the completely symmetric tensor of rank three Ψ^{ABC} in the six-dimensional space[7,8]:

$$\Psi^{ABC} \equiv \Psi^{\alpha i, \beta j, \gamma k} = D^{\alpha\beta\gamma, ijk} + \tag{13}$$
$$+ \frac{1}{3\sqrt{2}} [\epsilon^{\alpha\beta\delta} (2\epsilon^{ij} N^{\gamma,k}_{\delta} + \epsilon^{jk} N^{\gamma,i}_{\delta}) + \epsilon^{\beta\gamma\delta} (\epsilon^{ij} N^{\alpha,k}_{\delta} + 2\epsilon^{jk} N^{\alpha,i}_{\delta})]$$

Here α, β, γ and i,j,k are the unitary and spin indices, and $D^{\alpha\beta\gamma,ijk}$ and $N^{\alpha,i}_{\beta}$ describe the decuplet and the octet^x) of baryons, correspondingly.

The direct product $56^* \times 56$ splits into the following irreducible representations

$$56^* \times 56 = 1 + 35 + 405 + 2695 \tag{14}$$

Therefore, the amplitude is determined, just as in the previous case, by four invariants and can be written in the form of (3) with the condition (4) following from the Pauli principle. The amplitudes of different processes can be singled out from the expression of the type (3) with the aid of the wave function (13). The results for the scattering in the S-state $(c = -b,\ d = -a)$ are listed in the Table.

As is seen from the Table, $SU(6)$-symmetry imposes very severe restrictions on the amplitudes of baryon-baryon scattering and leads to an appearance of a great number of relations between the amplitudes of different processes in the $^1S_0$ and $^3S_1$ states^xx). Like in case of $SU(4)$ symmetry one cannot expect that these relations will be fulfilled in the region of very low energies. The mearge data available now are related just to the low energy region. In particular, the scattering lengths in Λp - collisions are likely to manifest strong spin dependence.

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x) In order to describe the baryon octet we make use of the traceless tensor N^{α}_{β} whose components are connected with the baryon states in the following manner $N^1_3 = p$, $N^2_3 = n$, $N^1_2 = \Sigma^+$, $N^2_1 = \Sigma^-$, $N^3_1 = \Xi^-$, $N^3_2 = -\Xi^0$,
$N^1_1 = \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda$, $N^2_2 = -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda$, $N^3_3 = -\frac{2}{\sqrt{6}}\Lambda$

xx) After this work had been finished we got preprint by M. Suzuki[16] concerning baryon-baryon scattering in $SU(6)$ - symmetry. The results tabulated in Table I of his paper seem to be incorrect.

Table

Matrix elements $\langle B_1(f)\, B_2(f)\, |T|\, B_1(i)\, B_2(i) \rangle$ of the reaction $B_1(i) + B_2(i) \to B_1(f) + B_2(f)$ in ${}^1S_0$ and ${}^3S_1$ states

$\langle B_1(f)\, B_2(f) \mid T \mid B_1(i)\, B_2(i) \rangle$	${}^1S_0$	${}^3S_1$
$\langle pp \mid T \mid pp \rangle$	$2(a - \frac{b}{27})$	0
$\langle np \mid T \mid np \rangle$	$a - \frac{b}{27}$	$a - \frac{b}{27}$
$\langle \Sigma^+ p \mid T \mid \Sigma^+ p \rangle$	$a - \frac{b}{27}$	$a + \frac{7b}{27}$
$\langle \Sigma^- p \mid T \mid \Sigma^- p \rangle$	$a + \frac{5b}{27}$	$a + \frac{7b}{81}$
$\langle \Lambda p \mid T \mid \Lambda p \rangle$	a	a
$\langle \Sigma^0 n \mid T \mid \Sigma^- p \rangle$	$-\frac{\sqrt{2}}{9} b$	$\frac{7\sqrt{2}}{81} b$
$\langle \Sigma^+ n \mid T \mid \Lambda p \rangle$	$-\frac{1}{9}\sqrt{\frac{2}{3}}\, b$	$\frac{1}{27}\sqrt{\frac{2}{3}}\, b$
$\langle \Sigma^0 p \mid T \mid \Lambda p \rangle$	$-\frac{1}{9\sqrt{3}} b$	$\frac{1}{27\sqrt{3}} b$
$\langle \Xi^- p \mid T \mid \Xi^- p \rangle$	$a + \frac{b}{27}$	$a + \frac{5b}{81}$
$\langle \Xi^0 n \mid T \mid \Xi^- p \rangle$	$\frac{4}{27} b$	$\frac{2}{81} b$
$\langle \Xi^0 p \mid T \mid \Xi^0 p \rangle$	$a + \frac{5}{27} b$	$a + \frac{7}{81} b$

Finally, we present some relations between the total cross sections resulting from the requirements of $SU(6)$ - invariance of the forward scattering amplitude and the optical theorem

$$3[2\sigma(\Lambda p) - \sigma(\Sigma^- p)] = 4\sigma(np) - \sigma(\Sigma^+ p) \tag{15.1}$$

$$\sigma(\Lambda p) + \sigma(\Xi^- p) - 2\sigma(\Sigma^- p) = \frac{1}{4}(\sigma(np) - \sigma(\Sigma^+ p)) \tag{15.2}$$

$$\sigma(\Sigma^- p) = \sigma(\Xi^0 p) \tag{15.3}$$

We express our sincere gratitude to Nguyen Van Hieu, V.I.Ogievetsky and Ya. A. Smorodinsky for useful discussions of the problems treated here.

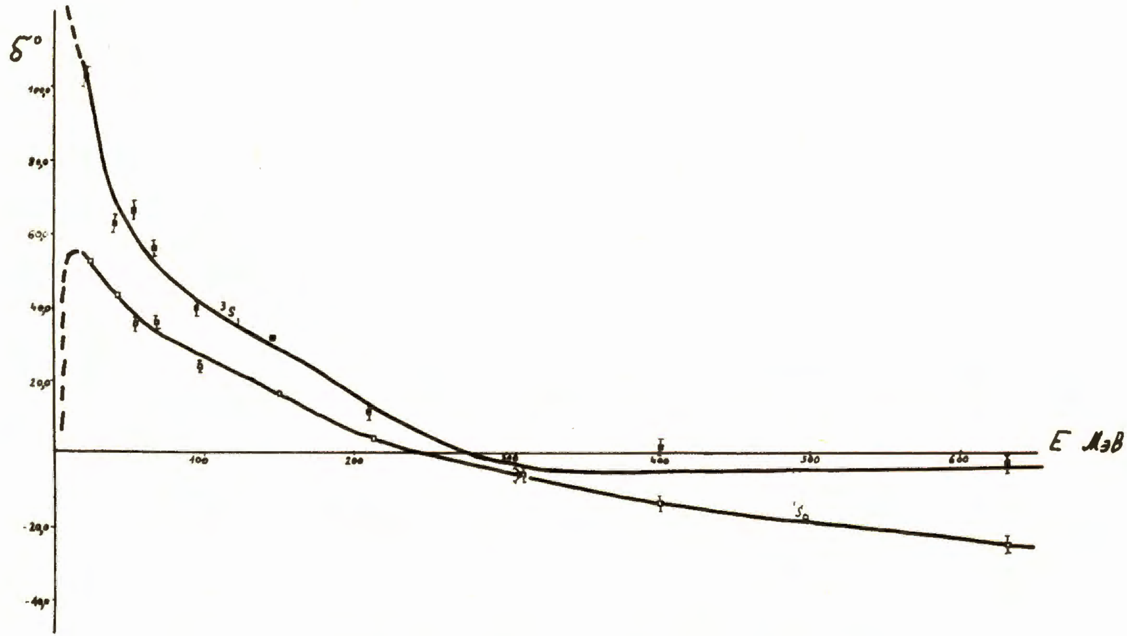

Fig.1. Energy dependence of $^{1}S_{0}$ and $^{8}S_{1}$ phase shifts.

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Received by Publishing Department
on May 3, 1965.