CHEBYSHEV POLYNOMIALS

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In memory of recently departed friends
Geoff Hayes, Lev Brutman

Preface

Over thirty years have elapsed since the publication of Fox & Parker's 1968 text *Chebyshev Polynomials in Numerical Analysis*. This was preceded by Snyder's brief but interesting 1966 text *Chebyshev Methods in Numerical Approximation*. The only significant later publication on the subject is that by Rivlin (1974, revised and republished in 1990) — a fine exposition of the theoretical aspects of Chebyshev polynomials but mostly confined to these aspects. An up-to-date but broader treatment of Chebyshev polynomials is consequently long overdue, which we now aim to provide.

The idea that there are really four kinds of Chebyshev polynomials, not just two, has strongly affected the content of this volume. Indeed, the properties of the four kinds of polynomials lead to an extended range of results in many areas such as approximation, series expansions, interpolation, quadrature and integral equations, providing a spur to developing new methods. We do not claim the third- and fourth-kind polynomials as our own discovery, but we do claim to have followed close on the heels of Walter Gautschi in first adopting this nomenclature.

Ordinary and partial differential equations are now major fields of application for Chebyshev polynomials and, indeed, there are now far more books on 'spectral methods' — at least ten major works to our knowledge — than on Chebyshev polynomials *per se*. This makes it more difficult but less essential to discuss the full range of possible applications in this area, and here we have concentrated on some of the fundamental ideas.

We are pleased with the range of topics that we have managed to include. However, partly because each chapter concentrates on one subject area, we have inevitably left a great deal out — for instance: the updating of the Chebyshev–Padé table and Chebyshev rational approximation, Chebyshev approximation on small intervals, Faber polynomials on complex contours and Chebyshev ($\mathcal{L}_\infty$) polynomials on complex domains.

For the sake of those meeting this subject for the first time, we have included a number of problems at the end of each chapter. Some of these, in the earlier chapters in particular, are quite elementary; others are invitations to fill in the details of working that we have omitted simply for the sake of brevity; yet others are more advanced problems calling for substantial time and effort.

We have dedicated this book to the memory of two recently deceased colleagues and friends, who have influenced us in the writing of this book. Geoff Hayes wrote (with Charles Clenshaw) the major paper on fitting bivariate polynomials to data lying on a family of parallel lines. Their algorithm retains its place in numerical libraries some thirty-seven years later; it exploits the idea that Chebyshev polynomials form a well-conditioned basis independent of the spacing of data. Lev Brutman specialised in near-minimax approximations and related topics and played a significant role in the development of this field.

In conclusion, there are many to whom we owe thanks, of whom we can mention only a few. Among colleagues who helped us in various ways in the writing of this book (but should not be held responsible for it), we must name Graham Elliott, Ezio Venturino, William Smith, David Elliott, Tim Phillips and Nick Trefethen; for getting the book started and keeping it on course, Bill Morton and Elizabeth Johnston in England, Bob Stern, Jamie Sigal and others at CRC Press in the United States; for help with preparing the manuscript, Pam Moore and Andrew Crampton. We must finally thank our wives, Moya and Elizabeth, for the blind faith in which they have encouraged us to bring this work to completion, without evidence that it was ever going to get there.

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John Mason
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April 2002

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Appendices: