

## CHAPTER 12

### Conclusion

In concluding a book that is at least as long as originally intended, we are painfully aware of all that we have left out, even though we have tried to include as much extra material in the Problem sets as we reasonably could. The reader is therefore referred to other texts for more advanced material, including the excellent book of Rivlin (1990), who includes fascinating material on various topics, such as the density of the locations of the zeros of Chebyshev polynomials of all degrees. We were also inspired by the book of Fox & Parker (1968), not only because it made the most up-to-date statement of its time but also, and even more, because it was extremely well written and stimulated major research. We hope that we are following in this tradition, and that there are numerous places in this book that future researchers might take as a starting point.

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